# *Elasto-Plastic Analyses of Circular Tunnels at Different Insitu Stress Conditions using Finite Element Method*


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## ABSTRACT

The design and construction of underground openings and their lining requires the knowledge of displacements and stresses in the surrounding rock due to excavation. For the displacement prediction to be of practical use, it should account for number of parameters, such as tunnel depth, diameter, construction procedure, initial state of stress, geological conditions, and strength behaviour of the rock mass. Analytical solutions considering elasto-plastic behaviour of the rock are based on simplifying assumptions and as such are not applicable to a general field situation. In this study, the finite element method has been used for elasto-plastic analyses of a circular tunnel using Hoek-Brown yield criterion with post peak softening behaviour. Three insitu stress ratios ($K_0$ = 0.5, 1.0, 1.5) have been considered. The deformed shape, yielded zone and stresses have been analyzed. The numerical results have been compared with analytical solutions for $K_0$ = 1.0. Effect of shotcrete lining was analyzed after release of three different percentages of excavation loads (25%, 50%, 75%) for three insitu stress ratios ($K_0$ = 0.5, 1.0, 1.5). It has been found that post peak softening affects significantly the extent of yielded zone, deformed shape and stress distribution. Ground reaction (response) curve developed from finite element method has been compared with ground reaction curve obtained from analytical solutions for $K_0$ = 1.0.



*Key Words:* Underground openings, elasto-plastic analysis, insitu stress, ground reaction curve, shotcrete lining.


## 1. NTRODUCTION

The analysis of geotechnical engineering problems is inherently complex in nature. There are many significant factors, which affect the behaviour of structures in or on the earth surface and are difficult to account for. Accordingly simplified assumptions have to be made, so that results with certain limited degree of accuracy are obtained. For design of underground openings, the resulting state of stress distribution and the displacements of various points around the openings due to excavation are required so that adequate support system can be designed. The stresses developed around an underground opening depend on a variety of factors, e.g., geologic conditions, shape and size of the opening, construction procedure, mechanical properties and insitu stresses of the media and the duration of time period during which the opening is left unsupported.

In the present study, a deep circular tunnel excavated in single stage has been analyzed. Hoek-Brown yield criterion has been used for the elasto-plastic analyses. A comparison of elastic and elasto-plastic analyses for single stage excavation has been carried to show clearly the differences and desirability of elasto-plastic analyses. Three insitu stress ratios ($K_0$=0.5, 1.0, 1.5) have been considered in the analyses. Effect of shotcrete lining was analyzed after release of three different percentages of excavation loads (25%, 50%, 75%) for three insitu stress ratios ($K_0$= 0.5, 1.0, 1.5). The deformed shapes and variation of principal stresses along some typical radial directions have been plotted. Ground reaction curve developed from finite element analyses has been compared with ground reaction curve obtained from analytical solutions for $K_0$=1.0.

## 2. ELSTO-PLASTICITY

The state of stress at a point is represented by the stress vector $\{\sigma\}$ ( $\{\sigma\}^T = [\sigma_x\ \sigma_y\ \sigma_z\ \tau_{xy}\ \tau_{yz}\ \tau_{zx}]$ ) and total strain vector $\{\varepsilon\}$ ( $\{\varepsilon\}^T = [\varepsilon_x\ \varepsilon_y\ \varepsilon_z\ \gamma_{xy}\ \gamma_{yz}\ \gamma_{zx}]$ ). In elasto-plastic state of the material behaviour, the total strain can be expressed as the sum of elastic and plastic strains, i.e.,

$$\{\varepsilon\} = \{\varepsilon^e\} + \{\varepsilon^p\} \tag{1}$$

where $\{\varepsilon^e\}$ and $\{\varepsilon^p\}$are the elastic and plastic strain vectors respectively.

The stresses are related to the total and plastic strains through the relation

$$\{\sigma\} = [D](\{\varepsilon\} - \{\varepsilon^p\}) \tag{2}$$

where [D] is the elasticity matrix. The elements of [D] are function of Young's modulus and Poisson's ratio.

In general the yield criterion can be expressed as

$$F(\{\sigma\}, \{\varepsilon^p\}, \overline{\lambda}) = 0 \tag{3}$$

where $\{\sigma\}$ is the stress vector, $\{\varepsilon^p\}$ is the plastic strain vector and $\overline{\lambda}$ is the hardening parameter. Equation 3 is known as yield function.

Depending on the value of F, the elastic or plastic state can be expressed as follows.

F < 0 : Elastic state
F = 0 : Plastic state

### *2.1 Flow Rule*

Since plastic deformation is a form of flow, the direction of the increment of plastic strain is defined by a flow potential to which the strain increment vectors are orthogonal. The flow rule can be written as,

$$\{d\varepsilon^p\} = \lambda \frac{\partial Q}{\partial\{\sigma\}} \tag{4}$$

where $\{d\varepsilon^p\}$ is the plastic strain increment vector; $\lambda$ is a non-negative quantity, and the Q is the plastic potential function.

The flow rule relating stress to deformation is known as 'the associated flow rule' ($Q \equiv F$), where as for non-associated flow rule ($Q \not\equiv F$).

### *2.2 Failure Criterion*

The empirical failure criterion as proposed by Hoek and Brown (1980) can be written as

$$\sigma_1 = \sigma_3 + (m\sigma_c\sigma_3 + s\sigma_c^2)^{1/2} \tag{5}$$

where $\sigma_1$ is the major principal stress at failure, $\sigma_3$ is the minor principal stress, $\sigma_c$ is the uniaxial compressive strength of the intact rock material, m and s are the constants which depend upon the properties of the rock and upon the extent to which it has been broken before being subjected to stresses $\sigma_1$ and $\sigma_3$.

The uniaxial tensile strength $\sigma_t$ of the specimen is given by substituting $\sigma_1$=0 in Eq. 5 and by solving the resulting quadratic equation for $\sigma_3$. Thus,

$$\sigma_t = \frac{1}{2}\sigma_c\left[m - \left(m^2 + 4s\right)^{1/2}\right] \tag{6}$$

Thus, Eq. 5 includes both, failure criterion of rock and limited tension together. Thus, a separate tension criterion is not required. Equation 5 can be written as

$$F = \sigma_1 - \sigma_3 - (m\sigma_c\sigma_3 + s\sigma_c^2)^{1/2} = 0 \tag{7}$$

Thus the original rock mass is linear elastic until F ≤ 0 and goes to plastic state when F > 0. For brittle rocks, once the rock mass starts yielding, immediately rock mass breaks and softening occurs as shown in Fig.1. Up to point 1, the yield criterion is given by Eq. 7 with parameters m and s and once stress reaches point 1, it results in broken rock mass in the yielded zone leading to softening and curve drops down to level 2 and the governing yield criterion becomes

$$F = \sigma_1 - \sigma_3 - (m_r \sigma_c \sigma_3 + s_r \sigma_c^2)^{1/2} = 0 \tag{8}$$

where $m_r$ and $s_r$ are the parameters for the broken rock mass (Hoek and Brown, 1980).

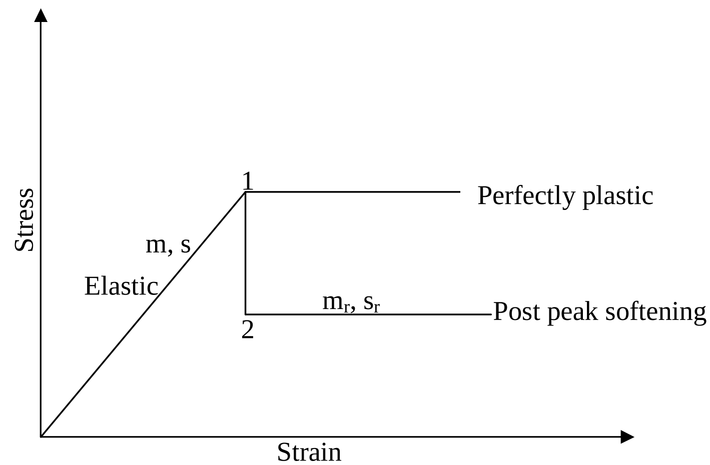



Fig. 1 – Idealized stress strain curve

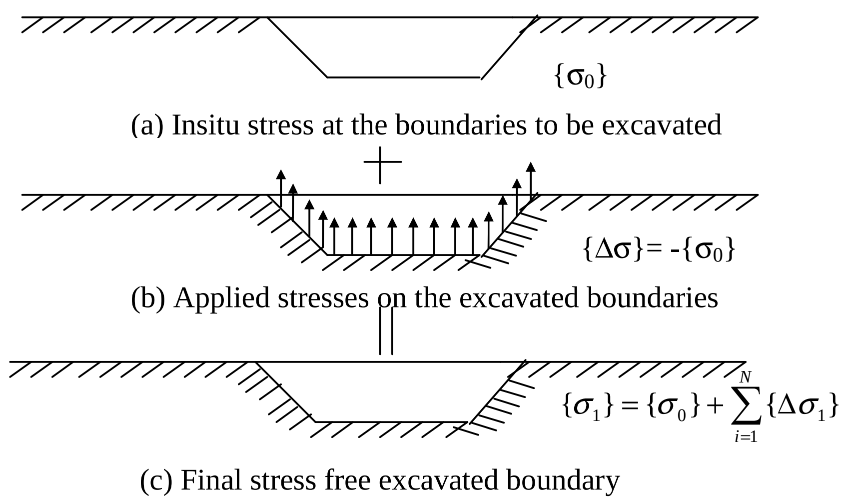



Fig. 2 – Simulation of excavation

## 3. SIMULATION OF EXCAVATION AND SHOTCRETE LINING

Excavation has been simulated in a single stage. Before the excavation is made, the stresses at any point in the medium are the insitu stresses due to overburden as shown in Fig.2a. After the excavation, the excavated boundary becomes stress free. This will be possible if stresses are equal and opposite of insitu stresses applied on the excavation boundary as shown in Fig. 2b. The influence of these applied stresses is considered by calculating the equivalent nodal loads for each element on the boundary of the excavation using the expression.

$$\{R_e\} = -\int_V [B]^T \{\sigma_0\} dV \tag{9}$$

where, $\{\sigma_0\}$ is the insitu stress. The element load vector $\{R_e\}$ thus calculated, forms right hand side of the Eq. 9.Details of excavation simulation are given by Sharma et al.(2001). This simulation procedure implemented by the second author in a computer program DSC-SST-2D (Desai, 1999) has been used for the analyses. Shotcrete lining has been simulated by adding elements at the boundary of the opening after the excavation. Eight-noded isoparametric elements have been used in the present study.

## 4. CASES ANALYSED

Analyses has been carried out for three different insitu stress ratios of $K_0 = 1$ (case 1), $K_0 = 0.5$ (case 2) and $K_0 = 1.5$ (case 3). For each case, a single stage excavation of deep circular tunnel of 10m diameter has been simulated considering post peak softening behaviour. Typical data from Hoek - Brown (1980) has been used in the analyses. Effect of shotcrete lining also has been considered. A schematic diagram (Fig. 3) shows the various cases and the analyses carried out. The properties of rock and shotcrete used in the analyses are as given below.

### *4.1 Rock Properties*

| | | |
|---|---|---|
| Rock type | = | Good quality gneiss |
| Young's modulus | = | 1400MPa |
| Poisson's ratio | = | 0.2 |
| Unconfined compressive strength | = | 70MPa |
| Vertical insitu stress | = | 3.5MPa |
| Material properties of original rock mass | = | $m = 0.5$, $s = 0.001$ |
| Material properties of broken rock mass | = | $m_r = 0.1$, $s_r = 0.0$ |

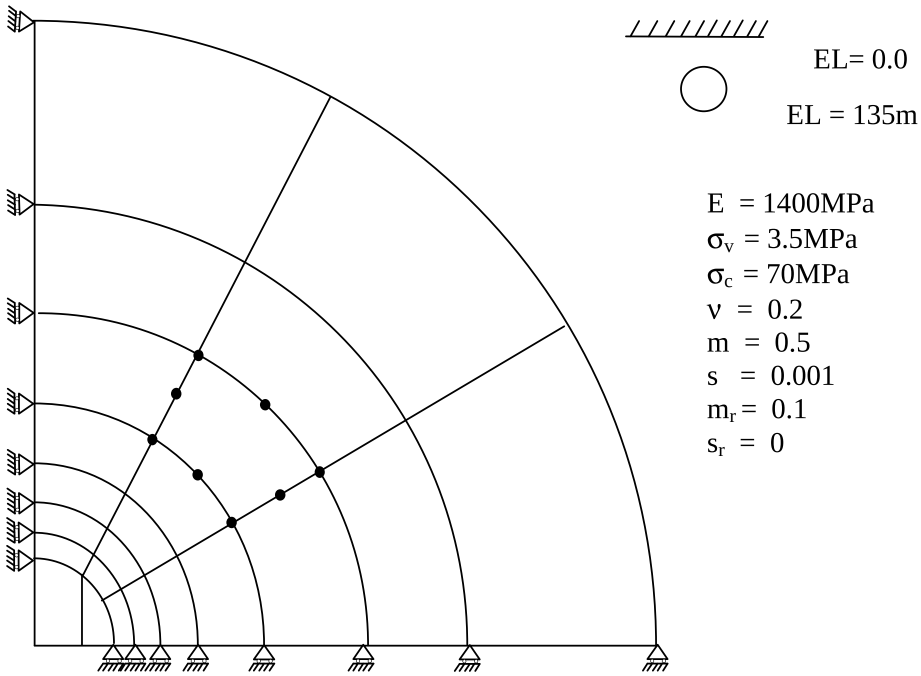
### *4.2 Shotcrete Properties*

| | | |
|---|---|---|
| Modulus of elasticity | = | 20.700GPa |
| Poisson's ratio | = | 0.25 |
| Compressive strength of shotcrete | = | 35MPa |
| Thickness of shotcrete layer | = | 0.05m |

## 5. DISCRETISATION AND ANALYSES

Taking symmetry of excavation into consideration, only quarter tunnel has been analysed. To simulate the infinite extent of the geological medium, the boundary elements are given as infinite elements. Close to the tunnel boundary where stress gradients change more rapidly, smaller size elements have been used and stress changes are less rapid further away, so increasingly larger elements have been used.

The quarter tunnel has been discretised into 3 segments (number of elements = 71, number of nodes =262). The discretisation is as shown in Fig. 4.




Total no. of elements = 71
Total no. of nodes = 262

Fig. 4 – Finite element mesh for circular tunnel

### *5.1 Case I ($K_0$ = 1.0)*

#### *5.1.1 Analyses without shotcrete lining*

Considering the intact rock properties (case 1, IRP, NL), the maximum deformation of 1.2cm is obtained at the boundary. Whereas considering the post peak softening (case 1, PPS, NL) the maximum deformation is 9.66 cm. In both the cases deformed shape of the tunnel is uniform and concentric. The stress contours for both the cases are concentric to the tunnel shape (i.e. uniform circular shape). The nature of curves obtained for elasto-plastic analyses is similar to that reported in literature (Obert and Duvall, 1967). The curve for minor principal stress starts from zero at the tunnel boundary and increases to nearly insitu state of stress. The curve for major principal stress starts from a high value (about one and half times the insitu stress value) and increases further (about three times the insitu stress value) and then decreases and finally reaches close to the insitu stress value after some distance (six to eight times the tunnel radius) from the tunnel boundary.

Figure 5 shows the variation of major and minor principal stresses along radial direction for the cases of post peak softening without lining. The displacements at various points along the radial line obtained by analytical and finite element methods are shown in Fig.6. It is seen from these figures that the two solutions compare very well. Ground reaction curves computed from finite element method and analytical methods are shown in the Fig.7. At no support, the displacement determined from analytical method is 120.6cm and from finite element method is 96.63cm at the tunnel boundary. It is seen from this graph the two solutions compare well.

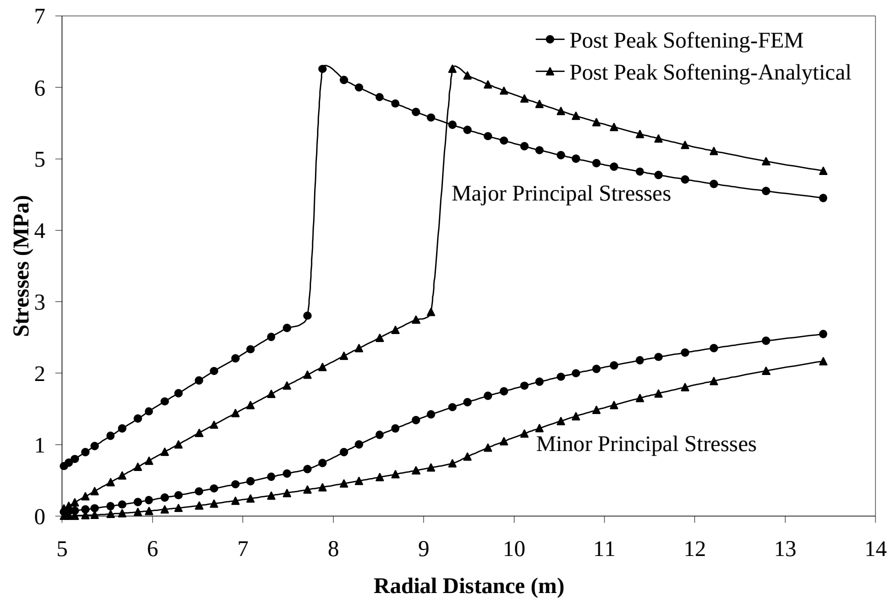



Fig. 5 – Principal stresses along the radial direction ($K_0$ = 1)

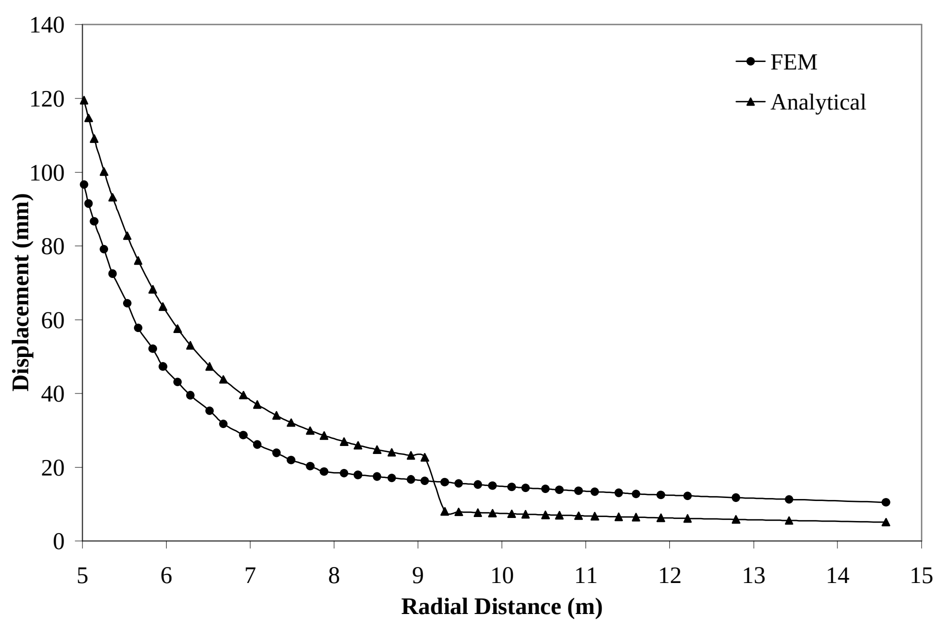



Fig. 6 – Displacement along the radial direction ($K_0$ = 1)

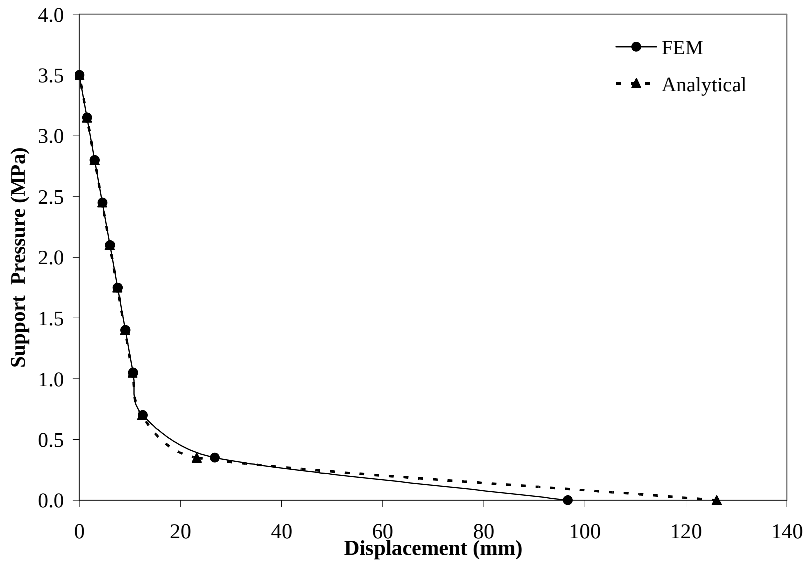



Fig. 7 – Ground reaction curve ($K_0$=1)

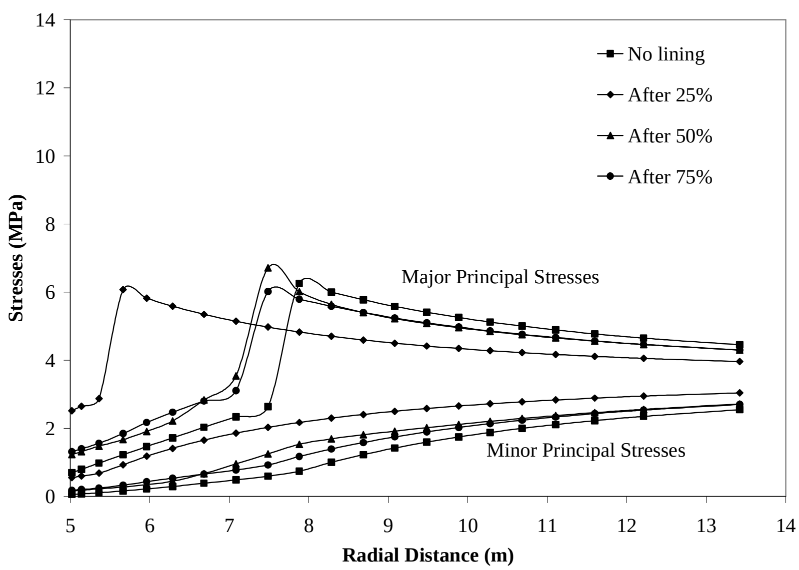



Fig. 8 – Principal stresses along the radial direction before and after the lining ($K_0$=1)

*5.1.2 Analyses with shotcrete lining*

The effect of shotcrete lining is considered by taking three different conditions. The maximum displacements occurred at boundary of the opening are as follows.

Case A) Support applied after 25% release of excavation load-1.70cm
Case B) Support applied after 50% release of excavation load-5.55cm
Case C) Support applied after 75% release of excavation load-5.93cm

For above cases deformed shape of the tunnel is uniform and concentric. Principal stresses along the radial direction for cases without and with lining are as shown in Fig.8. The thicknesses of the broken zones are presented in the Table 1.

Table 1 - Thickness of the broken zones ($K_0$ = 1.0)

| Cases Analysed | Thickness of Broken Zone (cm) |
|---|---|
| With intact rock properties | |
| No shotcrete lining | 96.0 |
| Post peak softening | |
| Support applied after 25% release of excavation load | 36.4 |
| Support applied after 50% release of excavation load | 91.9 |
| Support applied after 75% release of excavation load | 131.3 |
| No shotcrete lining | 171.6 |

## *5.2 Case 2 ($K_0$ = 0.5)*

### *5.2.1 Analyses without shotcrete lining*

The maximum deformation (1.96cm) is obtained at the crown of the opening and minimum deformation at the springing of the tunnel with intact rock properties (case 2, IRP, NL). With post peak softening (case 2, PPS, NL), the deformation near the springing (20.04cm) increases markedly as compared to those obtained with intact rock properties. In both the cases the deformed shape is not concentric. The stress contours for both the cases are not concentric to the tunnel shape. The curve for minor principal stress starts from a very low value near the tunnel boundary and increases to nearly insitu state of stress. The major principal stress with intact rock properties starts from a high value (8.566MPa) and decreases gradually and finally reaches close to insitu stress value. The major principal stress with post peak softening starts with a lower value (0.325MPa) and rises suddenly (8.563MPa) at the boundary of plastic zone (at radius of 9.479m) and then again decreases and finally reaches close to the insitu stress value.

### *5.2.2 Analyses with shotcrete lining*

The maximum displacements occurred at the springing are as follows.

Case A) Support applied after 25% release of excavation load-7.80cm
Case B) Support applied after 50% release of excavation load-8.01cm
Case C) Support applied after 75% release of excavation load-8.35cm

For the above cases the deformed shape of the tunnel is not concentric. The principal stresses before and after the lining near springing are as shown in Fig. 9.

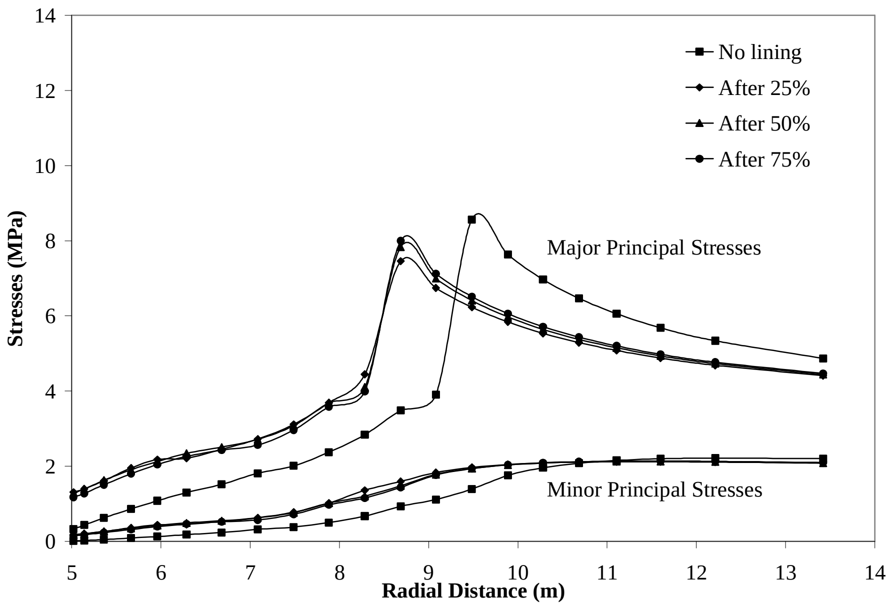



Fig. 9 – Stresses along the radial direction before and after the lining near the springing ($K_0$ = 0.5)

### *5.3 Case 3 ($K_0$ = 1.5)*

#### *5.3.1 Analyses without shotcrete lining*

With intact rock properties (case 3, IRP, NL) the maximum displacement (2.71cm) is obtained at the springing of the tunnel and minimum at crown of the tunnel. With post peak softening behaviour (case 3, PPS, NL), the deformation near the crown (36cm) of the tunnel increases markedly as compared to those obtained from intact rock properties. In both the cases the deformed shape is not concentric. The stress contours for both the cases are not concentric to the tunnel shape. The curve for minor principal stress starts from a very low value near the tunnel boundary and increases to nearly insitu state of stress. The major principal stress with intact rock properties starts from a high value (12.052MPa) and decreases gradually and finally reaches close to the insitu stress value.  The major principal stress with post peak softening behaviour starts with a lower value (0.670MPa) and rises suddenly (11.760MPa) at the boundary of plastic zone (at the radius of 11.599m) and then again decreases and finally reaches close to the insitu stress value.

#### *5.3.2 Analyses with shotcrete lining*

The maximum displacements occurred at the crown are as follows.

Case A) Support applied after 25% release of excavation load-14.60cm
Case B) Support applied after 50% release of excavation load-14.90cm
Case C) Support applied after 75% release of excavation load-15.70cm

For the above cases the deformed shape of the tunnel is not concentric. The comparison of maximum displacements for above all the cases is presented in Table 2.The comparison of yielded zones for the three insitu stress cases are presented in Table 3. The principal stresses before and after the lining near crown are as shown in Fig.10.

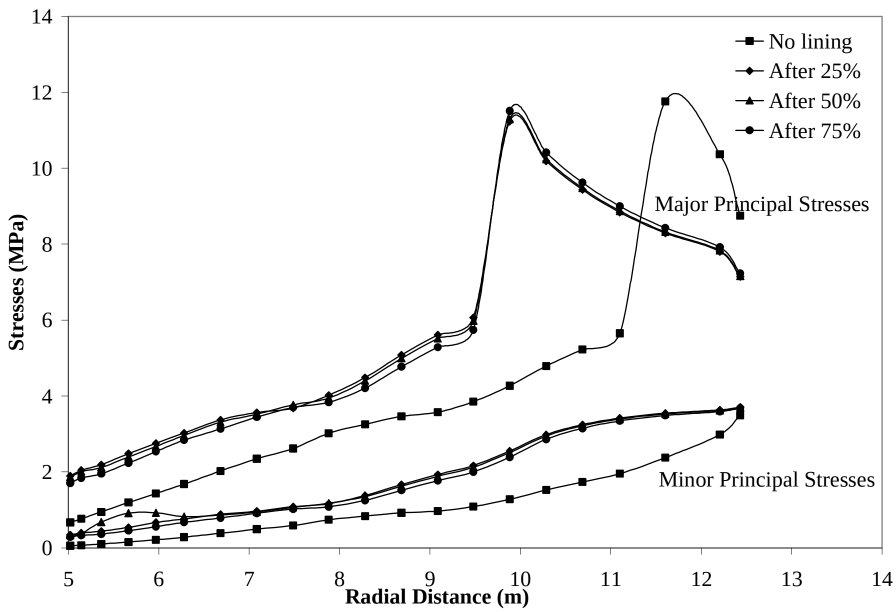



Fig. 10 – Stresses along the radial direction before and after the lining near the crown ($K_0$ = 1.5)

Table 2 - Comparison of Maximum Displacements (cm)

| Sl.No. | Description | Maximum Displacements, cm | | |
|---|---|---|---|---|
| | | $K_0$ = 1.0 (Radially) | $K_0$ = 0.5 (Crown) | $K_0$ = 1.5 (Springing) |
| *Intact rock properties* | | | | |
| 1 | No shotcrete lining | 1.20 | 1.96 | 2.71 |
| *Post peak softening* | | | | |
| 2 | Support applied after 25% release of excavation load. | 1.70 | 7.80 | 14.60 |
| 3 | Support applied after 50% release of excavation load | 5.55 | 8.01 | 14.90 |
| 4 | Support applied after 75% release of excavation load | 5.93 | 8.35 | 15.70 |
| 5 | No shotcrete lining | 9.66 | 20.40 | 36.00 |

Table 3 - Comparison of yielded zones (without lining)

| Insitu Stress Ratio | Yielded Zone (Radial distance) (m) |
|---|---|
| $K_0 = 1.0$ | 7.881 (radial) |
| $K_0 = 0.5$ | 9.485 (springing) |
| $K_0 = 1.5$ | 11.602 (crown) |

## 6. CONCLUSIONS

The displacements around an underground excavation depend mainly on insitu stress ratio ($K_0$). Maximum displacement noted at the boundary of excavation for stress ratio ($K_0$) other than 1 is compared to the displacements around the excavation for stress ratio ($K_0$) equal to 1.

With intact rock properties the maximum displacement is obtained at the springing if $K_0$ is greater than one and near the crown if $K_0$ less than one.

With post peak softening behaviour, the maximum displacement is obtained near the crown if $K_0$ is greater than one and near the springing if $K_0$ less than one.

Introduction of shotcrete lining as a support medium after release of 25% excavation load, displacements were reduced considerably, (9.66mm to 1.70mm at $K_0$=1.0, 20.40mm to 7.80mm at $K_0$=0.5, 36.00mm to 14.60mm at $K_0$=1.5). Effect of shotcrete is more effective, when it is installed after release of low percentage of excavation load, than installed after release of higher percentage of excavation loads for hydrostatic condition ($K_0$=1.0).

It has been found that deformation around the opening is governed by insitu stress ratio. Post peak softening behaviour affects significantly the extent of yielded zone, deformed shape and stress distribution.

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