Least Square Support Vector Machine Applied to Elastic Modulus of Jointed Rock Mass

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ABSTRACT

This paper uses least square support vector machine (LSSVM) for the determination of elastic modulus (E_j) of jointed rock mass. LSSVM is firmly based on the theory of statistical learning, uses regression technique. The inputs of the LSSVM model are joint frequency (J_n), joint inclination parameter (n), joint roughness parameter (r), Elastic modulus of intact rock (E_i) and confining pressure (? ${}_3$). LSSVM has been used to compute error barn of predicted data. An equation has been developed for the determination of E_j of jointed rock mass. Sensitivity analysis has been also performed to investigate the importance of each of the input parameters. A comparative study has been presented between LSSVM and artificial neural network (ANN) model. This study shows that LSSVM is a powerful tool for determination E_j of jointed rock mass.

***Keywords**:* Least square support vector machine; Elastic modulus; Jointed rock mass; Joint factor; Artificial neural network; Sensitivity analysis

1. INTRODUCTION

Elastic modulus (E_j) of jointed rock mass plays a major and crucial role for the design of civil engineering structure such as arch dams, bridge piers and tunnels. So, the determination of E_j of jointed rock mass is an imperative task in rock engineering. Researchers have used different empirical correlation for the prediction of E_j (Ramamurthy and Arora, 1994; Sitharam et al., 2001). Ramamurthy and Arora (1994) provided exponential relations to express the E_j of jointed rocks in terms of a joint factor. But, these empirical relations have some limitations (Maji and Sitharam, 2008). The limitations of empirical relations have been overcome by using Artificial Neural Network (ANN) (Maji and Sitharam, 2008). But, ANN has also some limitations such as arriving at local minima, less convergence speed, black box approach, less generalization performance and absence of probabilistic output (Park and Rilett, 1999; Kecman, 2001). This paper examines the potential of Least Square Support Vector Machine (LSSVM) for the prediction of E_j of jointed rock mass from the elastic modulus (E_i) of intact rocks and different joint parameters for various different confining pressure conditions. The important joint parameters which are taken into consideration independently are joint frequency (J_n), joint inclination parameter (n) and joint roughness parameter (r). The LSSVM is a statistical learning theory which adopts a least squares linear system as loss functions instead of the quadratic program in original support vector machine (SVM) (Suykens at al., 1999). LSSVM is closely related to regularization networks (Smola, 1998). With the quadratic cost function, the optimization problem reduces to finding the solution of a set of linear equations. The paper has the following aims:

- To determine the feasibility of LSSVM model for the prediction of Ej of jointed rock mass
- To make a comparison between developed LSSVM model and ANN model developed by Maji and Sitharam(2008)
- To compute the error bar of the predicted data
- To develop an equation for the prediction of Ej of jointed rock mass
- To do sensitivity analysis for determining the effect of different input parameters

2. LSSVM MODEL

LSSVM models are an alternate formulation of SVM regression (Vapnik and Lerner, 1963) proposed by Suykens et al (2002). Consider a given training set of N data points $\{x_k, y_k\}_{k=1}^{N}$ with input data $x_k \in R^N$ and output $y_k \in r$ where R^N is the N-dimensional vector space and r is the one-dimensional vector space. The four input variables used for the LSSVM model in this study are E_i, J_n, r, n, and confining pressure (σ_3). The output of the LSSVM model is E_j. So, in this study, $x = [E_i, J_n, r, n, \sigma_3]$ and $y = E_j$. In feature space LSSVM models take the form

$$y(x) = w^T \varphi(x) + b \tag{1}$$

Where the nonlinear mapping $\varphi(.)$ maps the input data into a higher dimensional feature space; $w \in R^n$; $b \in r$; w = an adjustable weight vector; b = the scalar threshold. In LSSVM for function estimation the following optimization problem is formulated:

Minimize: $\frac{1}{2} w^T w + \gamma \frac{1}{2} \sum_{k=1}^{N} e_k^2$

Subject to: $y(x) = w^T \varphi(x_k) + b + e_k$, k=1,…,N. (2)

The following equation for E_j prediction has been obtained by solving the above optimization problem (Vapnik, 1998; Smola and Scholkopf, 1998).

$$E_j = y(x) = \sum_{k=1}^{N} \alpha_k K(x, x_k) + b \tag{3}$$

The radial basis function has been used in this analysis. The radial basis function is given by

$$K(x_k, x_l) = \exp\left\{-\frac{(x_k - x_l)(x_k - x_l)^T}{2\sigma^2}\right\} \text{k,l=1,…,N} \qquad (4)$$

Where σ is the width of radial basis function.

3. DETAILS OF PRESENT ANALSYSIS

This study uses the above methodology for the determination of Ej of jointed rock mass**.** The data has been collected from the work of Maji and Sitharam (2008). The complete database comprised of 896 datasets. Out of that 515 datasets are with confining case and rest with unconfined case.

In carrying out the formulation, the data has been divided into two sub-sets: such as

(a) A training dataset: This is required to construct the model. In this study, 726 out of the 896 data are considered for training dataset.

(b) A testing dataset: This is required to estimate the model performance. In this study, the remaining 170 data are considered as testing dataset.

To train the LSSVM model, radial basis function has been used as a kernel function. The data has been scaled between 0 and 1 before being presented to the model. In training process, the value γ and σ have been chosen by trial-and-error approach. In this study, a sensitivity analysis has been done to extract the cause and effect relationship between the inputs and outputs of the LSSVM model. The basic idea is that each input of the model is offset slightly and the corresponding change in the output is reported. The procedure has been taken from the work of Liong et al. (2000). According to Liong et al. (2000), the sensitivity(S) of each input parameter has been calculated by the following formula

$$\text{S(\%)} = \frac{1}{\text{N}} \sum_{\text{j=1}}^{\text{N}} \left(\frac{\text{\% change in ouput}}{\text{\% change in input}}\right)_{\text{j}} \times 100 \qquad (5)$$

Where N is the number of data points. The analysis has been carried out on the trained model by varying each of input parameter, one at a time, at a constant rate of 20%. In the present study, training, testing and sensitivity analysis of LSSVM has been carried out by using MATLAB.

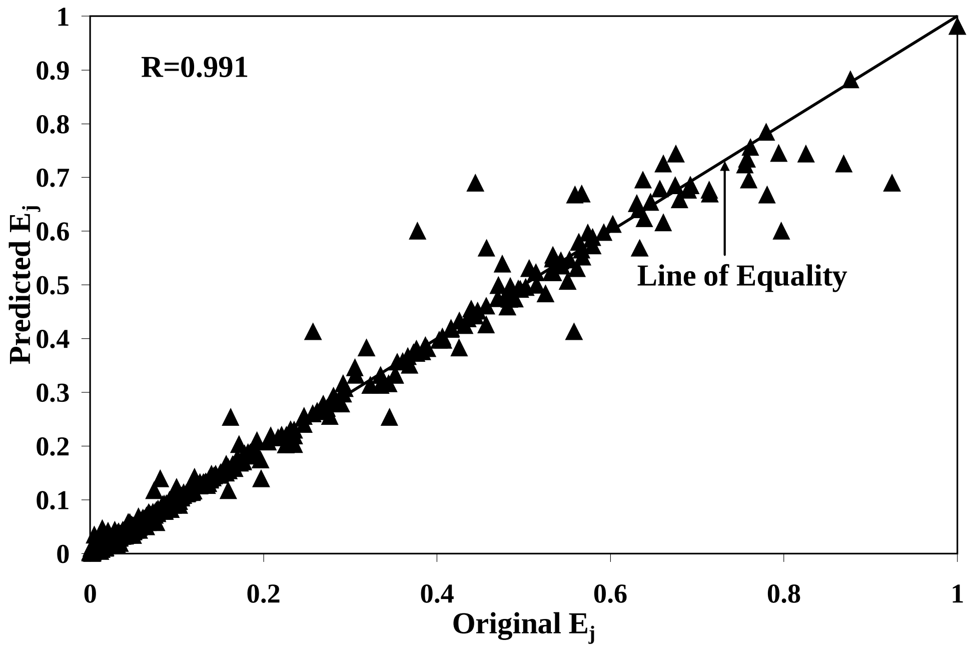


Fig. 1 - Performance of training dataset

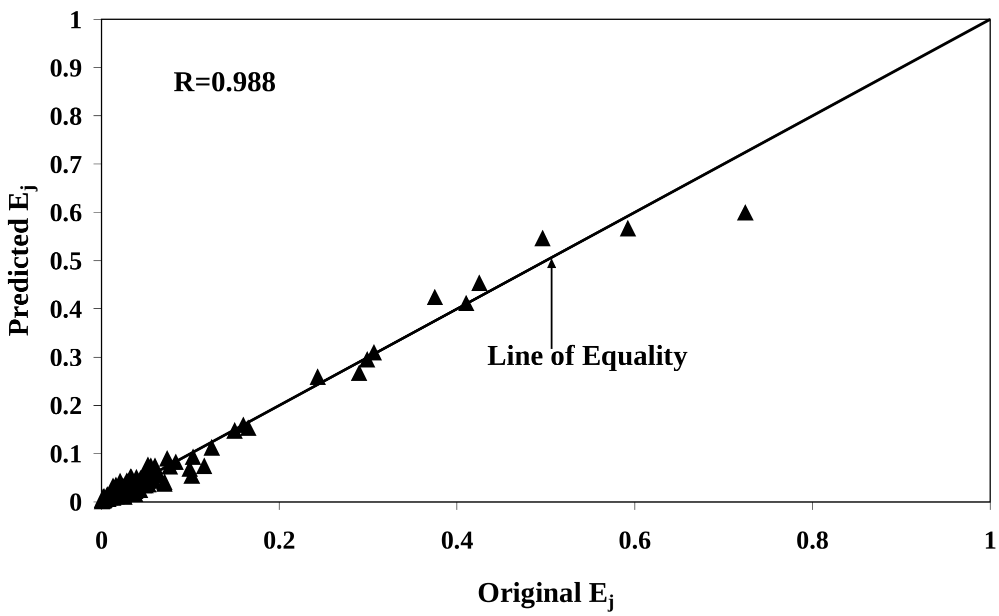


Fig. 2 - Performance of testing dataset

4. RESULTS AND DISCUSSION

The design value of γ and σ is 50 and 0.1 respectively. Figures 1 and 2 show the performance of training and testing dataset respectively. The loss of performance with respect to the testing set addresses generalization capability LSSVM's susceptibility to overtraining. There is a marginal reduction in performance on the testing dataset [i.e., there is a difference between SVM performance on training {coefficient of correlation (R) = 0.991} and testing (R) = 0.988)] for the LSSVM model. So, LSSVMs have the ability to avoid overtraining, and hence it has good generalization capability for the prediction of E_j.

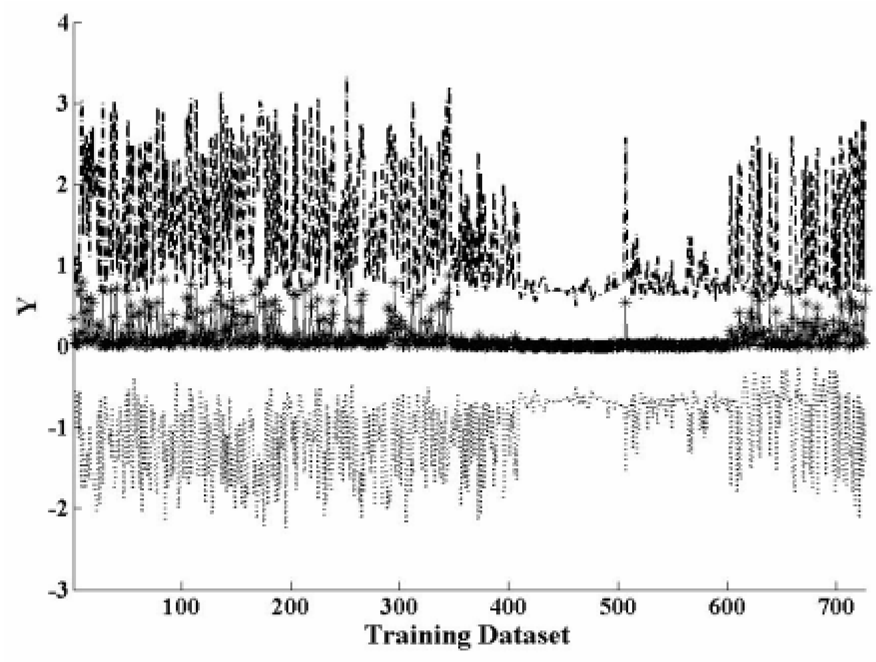
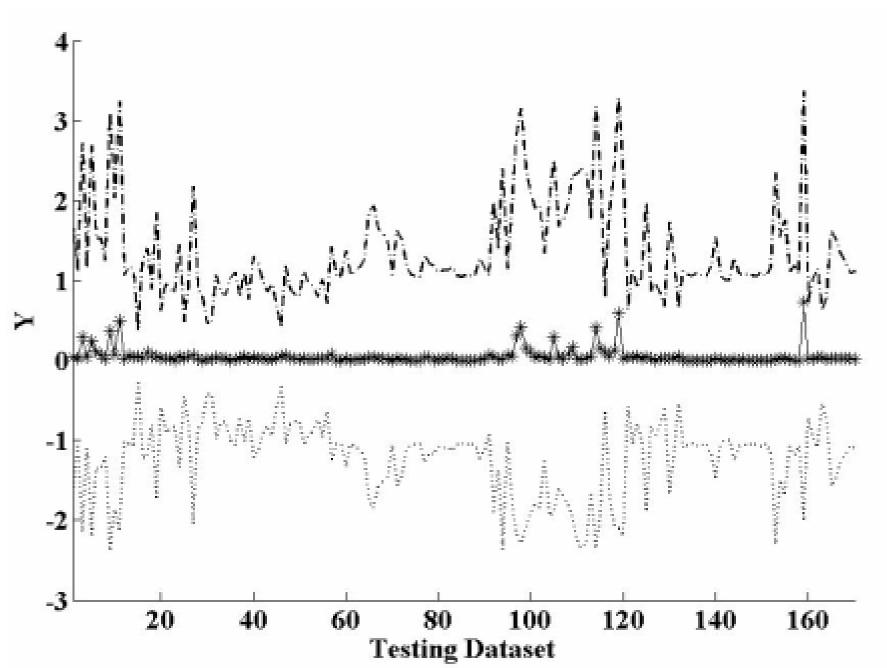


Fig. 3 - 95% error bar for training dataset.



Figures 3 and 4 show the 95% error bar of training and testing data respectively. So, the developed LSSVM model can be used to determine the E_j of jointed rock mass. Error bars have been used to indicate the range of one standard deviation on one prediction. It can be also used to determine whether differences are statistically significant. For the prediction of E_j, the determination of error bars on the point prediction is important in order to estimate the corresponding risk. The following equation can be used for the prediction of Ej of jointed rock based on the developed LSSVM model (by putting the value of σ = 0.1 and b =0.6221 in Eq. 3).

$$E_j = \sum_{k=1}^{726} \alpha_k \exp\left\{-\frac{(x_i - x)(x_i - x)^T}{0.02}\right\} + 0.6221 \tag{5}$$

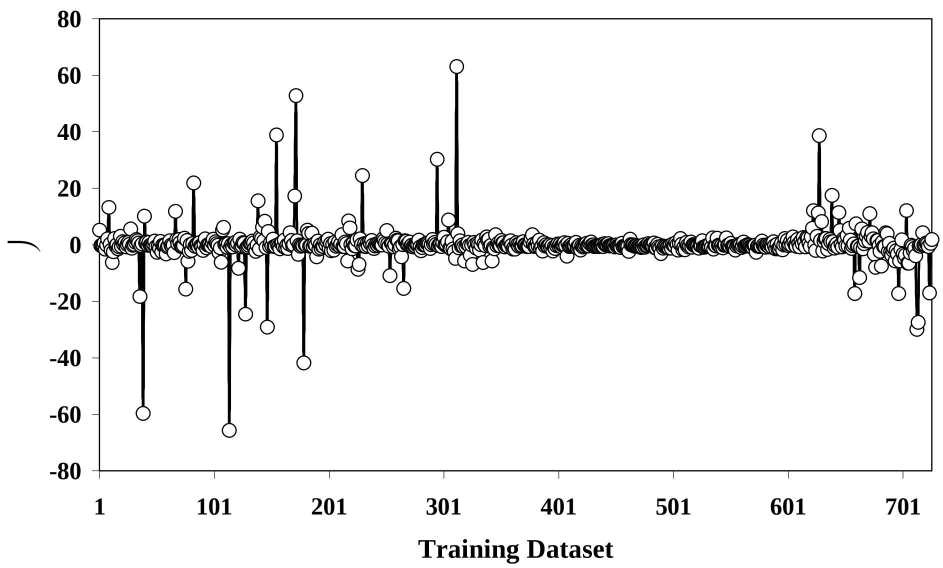


Fig. 5 - Values of α

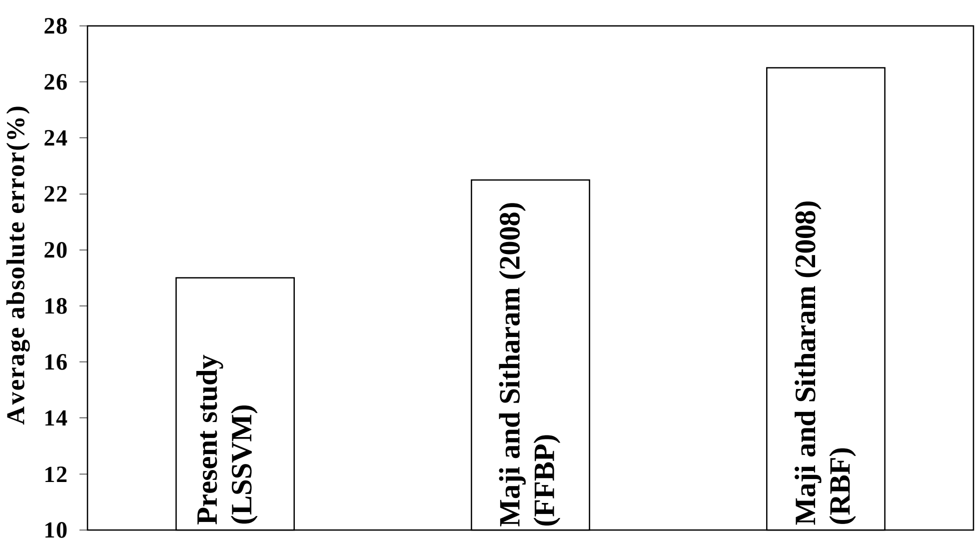


Fig. 6 - Comparison between LSSVM and ANN models.

The values of α are given by in Fig. 5. Figure 6 shows the comparison between developed LSSVM model and ANN models {feed forward back propagation (FFBP) and Radial basis function (RBF)} developed by Maji and Sitharam (2008). From the figure 6, it is clear that developed LSSVM model outperforms ANN models in terms of average absolute error (%). LSSVM uses mainly one kernel parameter. In ANN, there are larger number of controlling parameters, including the number of hidden layers, number of hidden nodes, learning rate, momentum term, number of training epochs, transfer functions, and weight initialization methods. Obtaining an optimal combination of these parameters is a difficult task as well. The result of sensitivity analysis is shown in Fig. 7. It can be seen that r has the most significant effect on the predicted E_j. Figure 7 also shows that σ_3 has the smallest impact on E_j. This point needs further research.

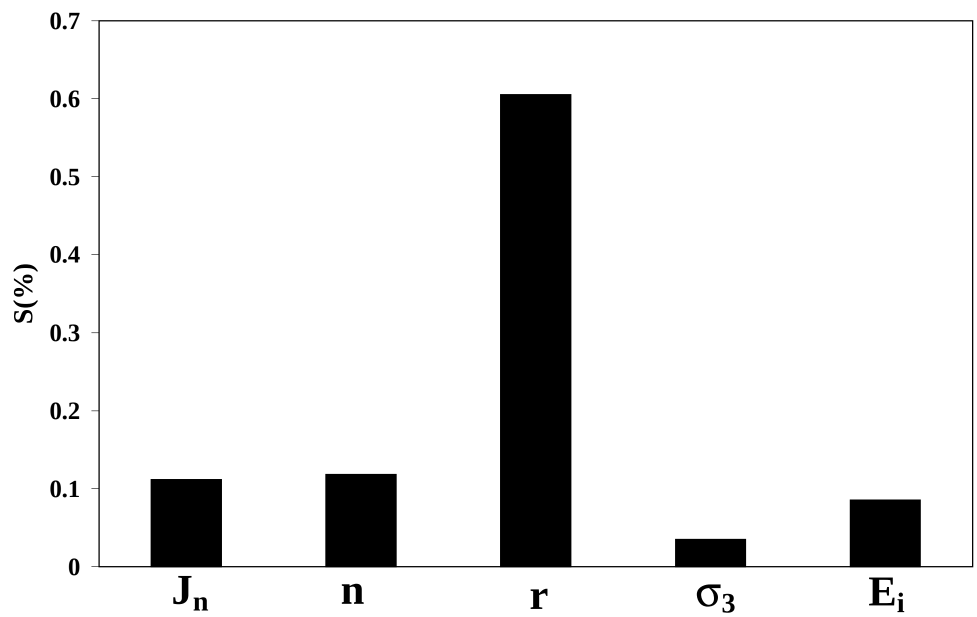


Fig. 7 - Sensitivity analysis of input parameters

5. CONCLUSIONS

This study examines the potential of LSSVM model for the determination E_j of jointed rock mass. LSSVM gives promising result for prediction of E_j of jointed rock mass and it also outperforms ANN models. The developed LSSVM model also gives error bar that yields confidence interval. Sensitivity analysis shows that r has the most significant effect on the predicted E_j followed by n, J_n, E_i and σ_3. Geotechnical engineers can use the developed equation for the prediction of E_j of jointed rock mass. The developed LSSVM model can be used as quick tool for the prediction of E_j of jointed rock mass without using any table or chart. This study gives a robust model based on LSSVM for the determination of E_j of jointed rock mass.

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