Simple Approach to Non-Linear Analysis of Jointed Rocks

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ABSTRACT

Most geotechnical problems are found to be non-linear and the rock mass behavior is highly complex and non-linear. For most of the civil/mining engineering applications involving jointed rocks we practically consider linear elastic constitutive behavior. Moreover the degree of non-linearity in rock mass depends on the joint system and also the degree of confinement makes the modelling very difficult. A simple approach for modelling the non-linear behavior of rock mass is presented here. The modified Hoek and Brown failure criterion and the confining stress dependent hyperbolic approach are used to capture the rock mass behavior very effectively. The model is implemented in finite difference based software program *FLAC* using *FISH* functions. Brief discussion on analysis and the procedure is presented here.

Keywords: Jointed rock; *FLAC*; Failure criterion; Non-linear analysis.

1. INTRODUCTION

The behaviour of rock mass by nature is non-linear and is distinguished from other engineering materials by the presence of inherent discontinuities such as joints, bedding planes and faults that control its behaviour (Fig. 1). Hence, the prediction of the response of rocks and rock masses derives largely from their discontinuous and variable nature. Overall, non-linear problem sources may be classified into two basic classes, first because of the non-linear constitutive law & the second due to the geometric nonlinearity giving rise to three type of problems, problem involving material non-linearity alone, problem involving geometric non-linearity alone and problem involving both material & geometric non-linearity. There are various solutions of nonlinear problems like incremental or stepwise procedure (Fig. 2a), iterative or Newton's method (Fig. 2b) and Step iterative or mixed procedure (Fig. 2c). The detailed literature regarding the different numerical procedure may be found in most of the related text books. It is important to understand the constitutive behaviour for the safe and economic design of civil and mining structures such as arch dams, bridge piers, tunnels, slopes and large caverns. But the realistic evaluation of the deformation behaviour of rock mass has theoretical and experimental difficulties due to their complex geometry and non-linear nature and therefore for many geotechnical applications we practically consider linear elastic constitutive behaviour. The highly complex and non-linear behaviour of rock mass depends on the degree of jointing and also on the degree of confinement. Therefore, a universal constitutive law may not be appropriate even if it is non-linear. A numerical approach to capture the behaviour with simple non-linear model with a capability of capturing different degrees with common formulation would be very helpful in this regards.

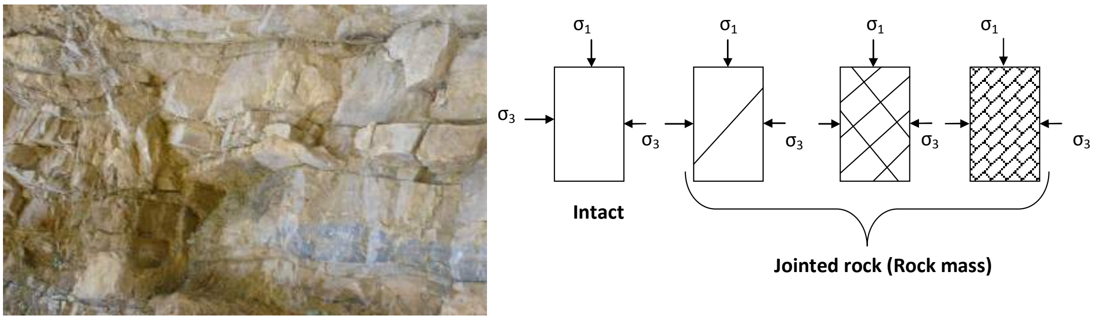
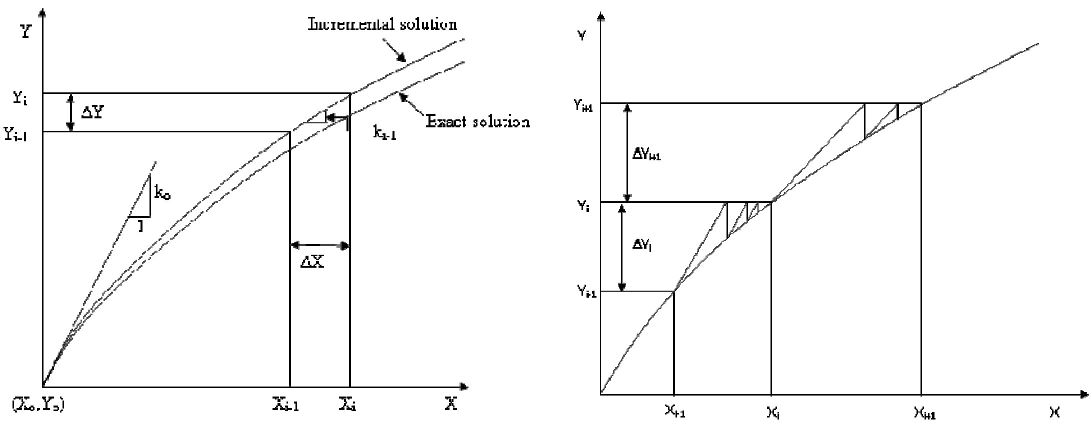
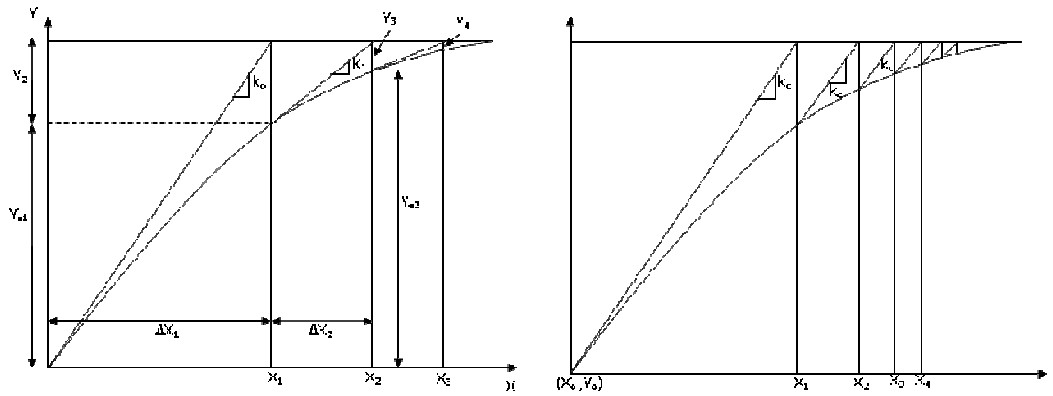


Fig. 1- (a) Rock mass encountered at the site (b) Representation of Intact rock and rock mass



(a) Incremental procedure (b) Step iteration or mixed procedure



(c) Iterative or Newton procedures

Fig. 2 - Different numerical procedure for non-linear analysis

Several models have been developed by various researchers to capture the non-linear behaviour of rock masses. Hoek and Brown (1980) arrived at an empirical criterion capable of modelling the highly non-linear relationship between the minor and major principal stresses at failure and also predicting the influence of rock mass quality on the strength. Later, Hoek et al. (2002) have modified their criterion and gave a generalized criterion in which the shape of the principal stress plot or the Mohr envelope could be adjusted by means of a variable coefficient 'a' in place of the square root term. Bieniawski (1974), Yudhbir et al. (1983) and Ramamurthy and Arora (1994) also developed some failure criteria to capture the non-linear stress-strain relationships of rock mass. Very recently, Singh et al. (2011) modified the Mohr-Coulomb criterion to find the non-linear triaxial and polyaxial strength of intact rocks.

The objective of the present paper is to know simple technique to capture the non-linear stress-strain behaviour of rock mass which are very complex and difficult to generalize while formulating. The implementation of the model using in-house/commercial software programs for easy and faster application may also need to be given due importance. Present paper deals with simple non-linear analysis of rock mass implemented in *FLAC*. The same can also be found in the proceedings of Indorock conference held during 13-15 October 2011 in IIT Roorkee, India.

2. NON-LINEAR BEHAVIOUR OF ROCK MASS

Stress-strain behaviour of rock mass or soil are non-linear although for modelling purposes elastic model mostly preferred because of its efficiency with numerical program. Non-linearity in the stress-strain behaviour of rock mass usually arises due to inherent discontinuities. In the excavation/construction process some new fractures/fissures/discontinuities are formed along with the widening of the existing fissures and fractures which leads to increase in the non-linearity of the behaviour. For a highly jointed rock mass, the non-linearity observed in the behaviour is also very high. As the number of joints decreases the behaviour of rock mass follow linearization process and for the intact rock with no joint the stress-stress behaviour is almost linear for most of the rocks. Non-linearity in the rock mass and stress-strain behaviour is also largely affected by confining stress condition. For the same joint configuration, with the increase in confining pressure the behaviour may change. Figure 3a depicts the hypothetical stress-strain behaviour from intact to highly jointed rock mass and Fig.3b shows the stress strain behaviour with varying confining conditions. For most cases, it has been observed that the initial tangent modulus increases with increasing confining stress. With increase in the confining stress the rock mass tends to be more reluctant to deform, as a result the deformation modulus increases. This stiffening phenomenon under confinement can be attributed mainly to the closure of joints, micro cracks, micro fractures, fissures etc. The nonlinearity can also be attributed through the concept of material degradation. Based on the experimental study by Fang and Harrison (2001), the degree of degradation displayed by failing rock reduces with increasing confining pressure, hence the nonlinearity in the behaviour. This stress dependent strength and deformation behaviour of rock mass is important for the safe and economic design of excavations in rock mass. In Fig. 3 it is shown that the, non-linearity of the stress-strain behaviour closely related with the nature and status of the rock mass. For a highly jointed rock mass, the non-linearity observed in the behaviour is also very high. As the degree of jointing decreases the behaviour of rock mass follow linearization process and for intact rock the stress-strain behaviour is almost linear for most of the rocks.

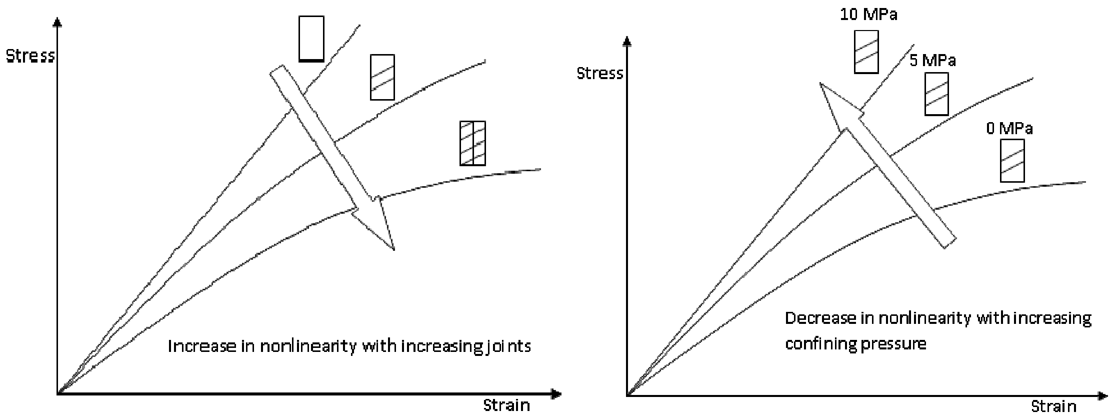


Fig. 3 - Hypothetical stress-strain behavior (a) from intact to highly jointed rock mass and (b) with varying confining conditions.

Non-linear behaviour of rock mass can be implemented with Hoek and Brown criterion or Hyperbolic model. Hoek and Brown (1980) introduced their failure criterion in an attempt to provide input data for the analyses required for the design of rock excavations in hard rock. The criterion started from the properties of intact rock and then introduced factors to reduce these properties on the basis of the characteristics of joints in a rock mass. Later introduction was made to a modified criterion to force the rock mass tensile strength to zero for very poor quality rock masses (Hoek et al., 1992). Figure 4 shows the Comparison of Mohr-Coulomb and Hoek and Brown envelope. The generalized Hoek-Brown criterion, represented by the equation:

$$\sigma_1' = \sigma_3' + \sigma_{ci}\left(m_b \frac{\sigma_3'}{\sigma_{ci}} + s\right)^a \tag{1}$$

Where, σ_1' and σ_3' are the major and minor effective principal stresses at failure; σ_{ci} the uniaxial compressive strength of the intact rock material; s is material constant, where s = 1 for intact rock; m_b is a reduced value of the material constant m_i and s and a are constants for the rock mass given by the following relationships:

$$m_b = m_i \exp\left(\frac{GSI-100}{28-14D}\right); \qquad s = \exp\left(\frac{GSI-100}{9-3D}\right); \qquad a = \frac{1}{2} + \frac{1}{6}\left(e^{-GSI/15} - e^{-20/3}\right) \tag{2}$$

The equivalent friction angle and cohesion can be calculated using the equations as presented in paper of Hoek and Brown (1997). The model can easily be implemented using *FLAC* or any other commercial packages. Hoek and Brown model can be implemented by modifying the inbuilt Mohr-Coulomb model by changing the cohesion and friction values continuously.

Increase in confining pressure usually results in a steeper stress-strain curve and a higher strength and the values of initial tangent modulus (E_i) and $(\sigma_1 - \sigma_3)_{ult}$ therefore increases with increasing pressure. This stress dependency can easily be taken into account by using empirical equations as given by Janbu (1963). The material behaviour of intact rock can be modelled using the nonlinear relationships of Duncan and Chang (1970). Non-linear nature of the rock mass may be captured using the Hyperbolic model where the the expression for instantaneous slope of the stress strain curve is the tangent modulus ‘ E_t ’ can be find out using the folllowing expression,

$$E_t = \left[1 - \frac{R_f (1-\sin\phi)(\sigma_1 - \sigma_3)}{2c\cos\phi + 2\sigma_3 \sin\phi}\right]^2 K.Pa\left(\frac{\sigma_3}{Pa}\right)^n \tag{3}$$

This equation has been used to calculate the approximate value of tangent modulus for any stress condtion ‘ σ_3 ‘ and $(\sigma_1 - \sigma_3)$ if the values of the parameters K, n, c, ϕ and R_f are known. Where, c and ϕ are the cohesion and angle of internal friction respectively and ‘Pa’ is the atmospheric pressure.

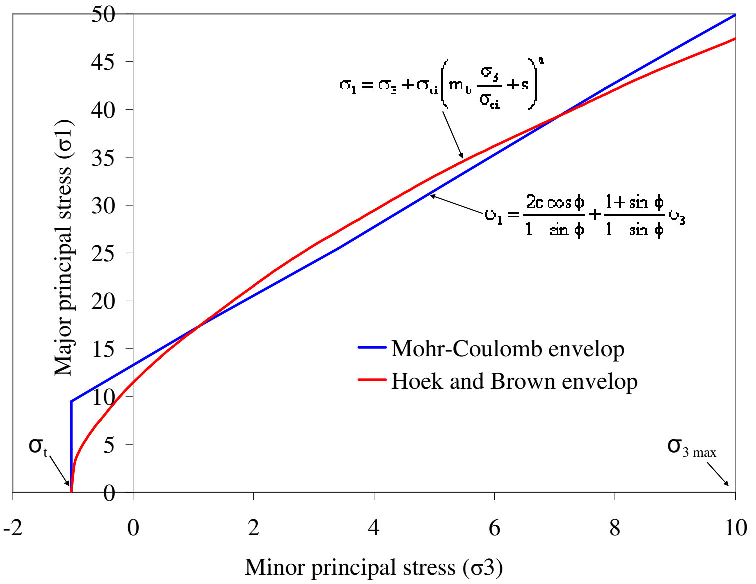


Fig. 4 - Comparison of Mohr-Coulomb and Hoek and Brown envelope

3. NUMERICAL IMPLIMENTATION

Implementation of the non-linear constitutive relationships into the numerical model is presented here. The incremental method is used for the solution of the non-linear problem. Implementation is done using explicit finite difference code, *FLAC* which is finite difference package from HC-ITASCA Consulting Group, Minneapolis, Minnesota, USA. Several built-in constitutive models are available in the code that permits the simulation of highly non-linear, irreversible response representative of geologic, or similar, materials. In addition, *FLAC* contains many special features like interface elements to simulate distinct planes along which slip and/or separation can occur, plane strain, plane stress and axisymmetric geometry modes, groundwater and consolidation, structural element etc. It also contains the powerful built-in programming language *FISH* with the help of which one can write one's own functions to extend *FLAC's* usefulness and even implement one's own constitutive models. In the present work, the model has been incorporated using *FISH* functions that modifies the built in constitutive models, Mohr-Coulomb and elastic to generaized Hoek and Brown and hyperbolic model respectively. Hyperbolic model calculates nonlinear elastic moduli as a function of confining pressure. Nonlinear stress or strain dependent material properties are implemented in *FLAC* by means of a *FISH* functions. The *FISH* function calculates nonlinear elastic moduli as a function of confining pressure. In order to implement a stress-dependent material property, the property adjustment is made at a regular interval of calculation step. In case of Hoek and Brown model the cohesion and friction values are updated. The *FISH* function is called at every 20th step, otherwise the function will be very slow in execution as it involves calculations for every zone. Therefore, it is necessary to control the step interval when the *FISH* function is to be called. A separate function 'supstep' is used to control the step interval when the parameter adjustment is made. The interval function is then invoked for a prescribed number of "super" steps with the function 'supsolve'. It is also critical that the adjustment of properties shouldn't produce a condition which is non-physical, and therefore while doing analysis, it is ensured that the property adjustment does not produce a condition which becomes numerically unstable. Numerical simulation of triaxial testing of intact and jointed rocks were done with 3-dimensional specimens having 76mm length and 38mm diameter have been considered similar to laboratory experiments. Figure 5 shows the finite difference mesh for cylindrical rock samples for both axi-symmetric and *FLAC3D*.

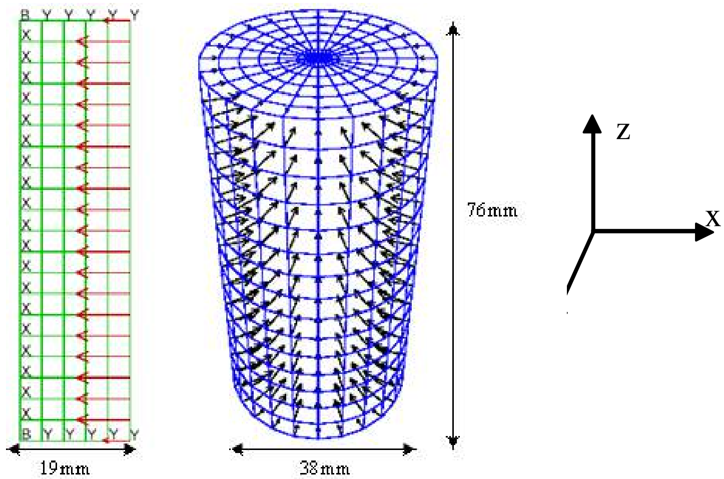


Fig. 5 - Cylindrical specimen model (Axi-symmetric model and *FLAC3D*)

The convergence of solution was decided based upon the unbalanced force which indicates a mechanical equilibrium state. When the model is in equilibrium, the net nodal force vector at each grid point is zero. The maximum unbalanced force will never exactly reach zero for a numerical analysis. The model is considered to be in equilibrium when the maximum unbalanced force is small compared to the total applied forces in the problem. If the unbalanced force approaches a constant nonzero value, it is assumed that the failure and plastic flow are occurring within the model. In the present

analysis of triaxial specimen testing of rock sample the maximum unbalanced force is recorded with number of steps/cycles to check the convergence.
The non-linearity in the stress-strain behaviour of jointed rocks is incorporated in the numerical model and to capture this stress-strain behaviour using *FLAC*, the incremental procedure is adopted, where change in loading is analyzed in series of steps or increments. At the beginning of new increment of loading an appropriate modulus value is selected for each element on the basis of values of strain in that element. Non-linear confining stress dependent stress-strain behaviour of geomaterials may be modelled using finite element or finite difference program by assigning different modulus values to each of the elements/zones into which the material is subdivided for purpose of analysis. The modulus value assigned to each zone is based upon the stresses in each zone. As the modulus values depend on the stresses and the stresses in turn depend on the modulus values, it is necessary to make repeated analyses to ensure that the modulus values and the stress conditions correspond for each zone of the system.

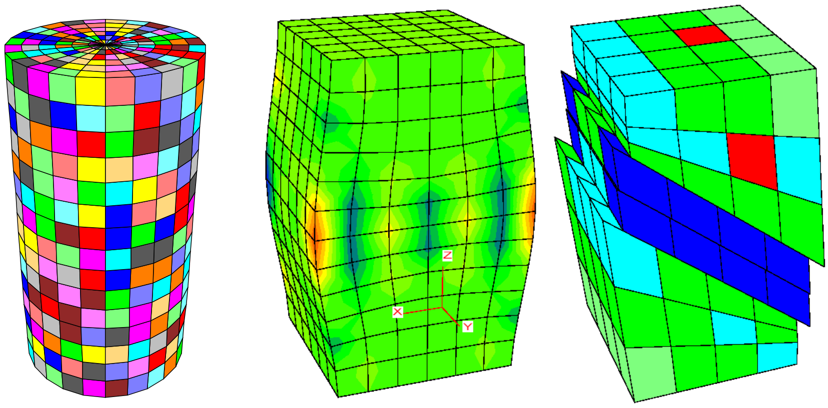
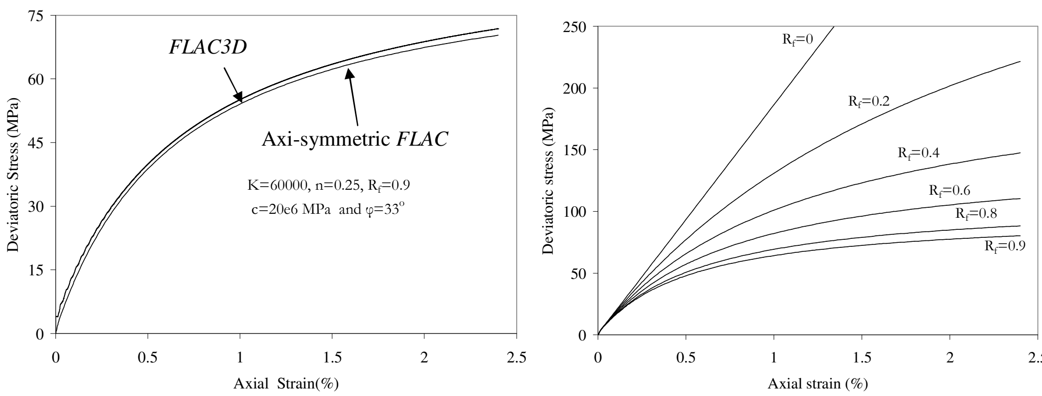


Fig. 6 - Non-linear analysis for the rock specimen test



(a) (b)

Fig. 7 - (a) Dependency of non-linearity of stress-strain behaviour with failure ratio (R_f) and (b) Stress-strain curve of specimen test

The non-linear stress-strain behaviour captured using *FLAC3D* and *FLAC*, of a jointed rock mass. Figure 6 shows how the different modulus values are assigned to a numerically simulated cylindrical specimen. The figure also shows the numerical testing block jointed specimen (Brown and Trollope, 1970) both as equivalent material and as explicit model. Figure 7a shows the stress-strain plot from *FLAC3D* and axi-symmetric analysis of cylindrical rock specimen test. Figure 7b shows the plot between deviated stress versus axial strain for different failure ratio (R_f) and depicts the efficiency of the numerical program in capturing the non-linear behaviour. The analysis can easily be extended to various field problems involving jointed rock mass. Figure 8 shows the comparison of equivalent material modelling results with the experimental values (Brown and Trollope, 1970) interpolated using cubic spline function, with $60^o/30^o$ block jointed specimen at four different confining pressures 1.4, 3.4, 6.9 and 13.8 MPa respectively.

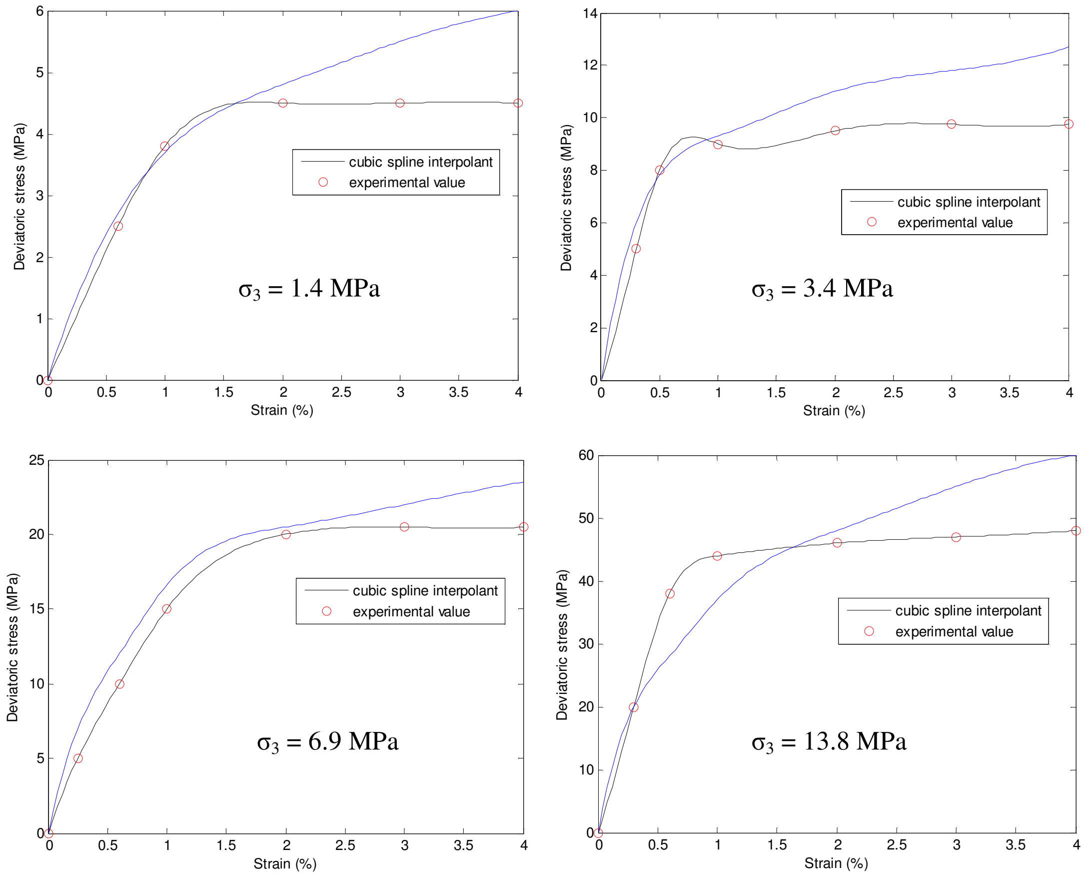
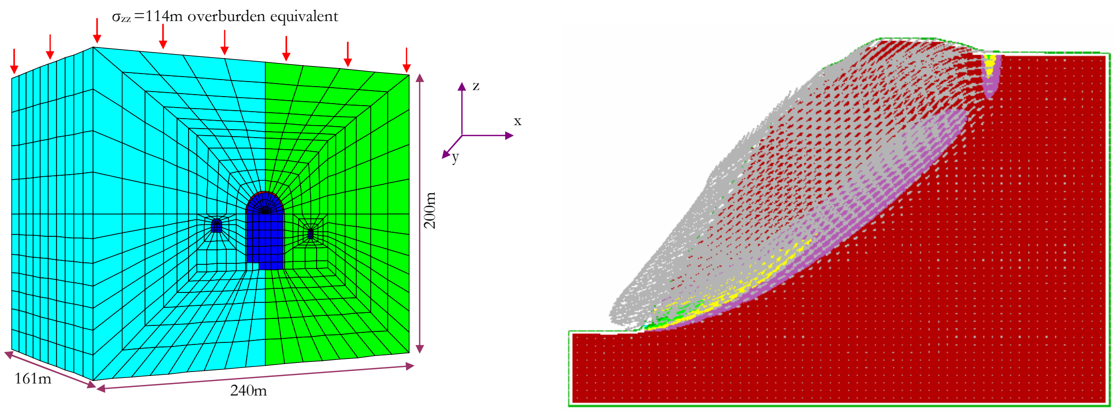


Fig. 8 - Comparison of equivalent continuum modelling results with the experimental values (Brown and Trollope, 1970) interpolated using cubic spline function, with $60^o/30^o$ block jointed specimen at four different confining pressures.

4. FIELD APPLICATIONS

Non-linear models can be easily adopted for applications to rock mass modelling for underground excavations and large scale slopes by investigating the stress and deformation behaviour. The model is verified by applying to underground power caverns and large scale slopes where the rock mass encountered is disturbed and non-linear models found to be more appropriate. Detailed on these analysis may be found elsewhere (Maji, 2007; Sitharam et al., 2007 and Maji & Sitharam, 2007). The 3-dimesional stress and deformational behaviour of the Shoibara power house cavern in Japan is analysed using *FLAC3D* using non-linear model (Fig. 9a). The results were compared with the instrumented data from the field. The modelling results were also compared with six other computation models as reported by Horii et al. (1999) which were used earlier to analyze the Shiobara power house cavern and it was observed that the results from the present work are comparable with the some well established complex equivalent continuum models.

A study was also conducted to analyze large scale slopes in jointed rock mass using the concept of non-linear equivalent continuum using *FLAC* (Fig. 9b). The stress and deformational study have been done for the original slope profiles together with the cut profiles with bridge (proposed to be constructed on this slope) pier loads. The critical failure surface with shear strain increment together velocity vector is shown in the Fig. 9b. The analysis found to be efficient in capturing the overall behavior of this large scale slope.



(a) (b)

Fig. 9 - (a) *FLAC3D* model of underground power cavern and (b) Critical failure surface for a large scale slope

5. CONCLUSIONS

A simple approach for performing the non-linear analysis of rock mass is demonstrated. Non-linear models, the Hoek and Brown and hyperbolic models are implemented in *FLAC* using *FISH* functions. The hyperbolic model with Mohr-Coulomb yielding gives a smooth transition between elastic to plastic behaviour and has the advantages that they can realistically model non-linear stress-strain behaviour of rocks/rock mass. However, the hyperbolic relationships are most suitable for analysis of stress and deformation behavior prior to failure. The Hoek and Brown model can suitably be applied to highly disturbed isotropic rock mass by choosing correct values of ‘m’ and ‘s’ parameters and adjusting the parameter ‘a’. The non-linear models can be applied to field problems with large scale failure and found to be very effective while predicting the strength and deformation behaviour. Mathematical formulation of this models are very simple and can easily be implimented in commercial programs.

Acknowledgements

The author thanks his Ph.D. supervisor at Indian Institute of Science, Bangalore, India Prof. T. G. Sitharam for his guidance, supports and encouragements. Major part of the work presented here are from his Ph.D. thesis.

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