Study of Numerical Modelling of a Rock Slope Based on Modified Mohr-Coulomb Criterion

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ABSTRACT

The slopes near highways on hilly region are vulnerable and require special attention. These slopes are analyzed extensively for potential failure as any movement in slope may cause significant damage to infrastructure and lives residing nearby it. This is achieved through extensive investigation and monitoring program supported by detailed study. However most input values measured in the field or obtained by laboratory tests which are used subsequently to analyze the problem show a wide scatter across a significant range rather than being a fixed single value. In the present study probabilistic analysis of the input parameters which are required for slope stability analysis has been done so as to arrive at a single representative value. The shear strength parameters have been found out following modified Mohr-Coulomb criterion. Numerical Analysis has been done using Finite Difference FLAC-2D software and Finite Element PHASE^2 software. Application of the numerical modelling enabled a representative simulation of observed slope conditions. The stability of slopes is further studied for limit equilibrium using computer programs developed for circular failure mechanism as continuum characterization of rock mass deemed to be reasonable.

Keywords: Numerical analysis; Shear strength; Modified Mohr-Coulomb criterion; Continuum modelling; Slope stability

1. INTRODUCTION

The stability evaluation of natural rock slope is essential especially when the slopes are situated close to residential areas or when important highways are built on these slopes. A rock slope failure is a very complex natural activity. The slope failure is governed by the presence of weakness or discontinuities. The persistence of key discontinuities leads to failure of rock slopes in one or a combination of several mechanisms like planar sliding, wedge sliding or toppling. However if the rock mass in slope is justified as homogeneous medium then the failure is said to be governed by circular failure mechanism. To assess the stability of a natural slope in jointed rock masses, all the possible failure modes have to be checked.

A variety of slope stability analysis methods had developed with passage of time. In last one century a significant number of geotechnical researchers have contributed for slope stability evaluation techniques. The slope stability evaluation methods can be broadly classified under following subheadings.

- Limit Equilibrium method
- Limit analysis method
- Numerical analysis methods
- Soft computing and natural algorithm techniques

A detail of these techniques had been critically reviewed by Nash (1987), Thompson (1993), Duncan (1996, 2000), Donald and Chen (1997), Zhang et al. (1997), Yu et al. (1998), Fredlund and Scoular (1999), Griffiths and Lane (1999), Cala and Flisiak (2003), Krahn (2003), Hammah et al. (2004), Saklereria and Frantau (2005), Li et al. (2008) and Pourkhasravani and Kalantari (2011).

The limit equilibrium and numerical analysis methods are still the best choices of engineers because of their easy applicability and fewer boundary conditions and assumptions. The limit equilibrium method does not consider stress strain relationship but it provides an estimate of factor of safety based on force and moment equilibrium of slope material above an assumed failure plane.

With the advancement in computational techniques, Numerical method proved as useful tool for slope stability analysis. The numerical analysis is further classified for continuum and discontinuum approximation of materials. When the slope material is justified as homogeneous and isotropic, continuum analysis is more reasonable. Finite element method (FEM) and Finite Difference method (FDM) are used for continuum numerical analysis.

The slope instability is mostly influenced by strength and geometrical parameters of slopes. The Limit equilibrium and Numerical analysis methods analyze the strength parameters and geometrical parameters of the rock mass to compute a parameter which defines the stability of slopes i.e. the factor of safety (FOS). The Factor of safety is defined as the ratio of reaction over action, expressed in terms of moments or forces, and eventually in term of stresses, depending upon geometry of assumed failure surface.

For numerical analysis, factor of safety is defined using gravity increase (GI) approach and shear strength reduction (SSR) approach. In the gravity increase approach, the gravity forces, such weight, increase gradually until the slope becomes unstable. The factor of safety of slope is defined as ratio between gravitational acceleration at failure time and actual gravitational acceleration. In the shear strength reduction approach, factor of safety is also defined as the ratio of actual shear strength to the minimum shear strength required to prevent failure. This is also known as shear strength reduction factor or SRF. Shear Strength Reduction technique is used with FEM and FDM to solve problem related to slope stability.

Most of the finite difference analyses of slopes are done using program FLAC. FLAC is widely used for analyzing stability of soil slopes (Zettler et al. 1999; Dawson and Roth 1999; Cala and Flisiak 2000, 2001, 2003). Sometimes FLAC is even used for slope stability engineering in combination with other methods. Thompson (1993) and Babu and Bijoy (1999) show examples of the application of FLAC combined with LEM. Sitharam and Maji (2007) and Latha and Garaga (2010) used FLAC for dynamic and static evaluation of highly jointed rock slope on the bank of Chenab river in Kashmir in India.

Due to its power and flexibility, the Finite Element Method (FEM) is increasingly being applied to slope stability analysis. Griffiths and Lane (1999) studied the advantage of using finite element method over other limit equilibrium method. Hammah et al. (2004) uses the Shear Strength Reduction (SSR) technique along with the FEM to calculate factors of safety for slopes. Various finite element packages were developed and used for slope stability analysis. Hammah et al. (2004) used Phase^2 software. Fang et al. (2010) used Ansys software to compute factor of safety of a slope section in open pit mine in china. Rentala and Satyam (2011) used Plaxis-2D software package to compute the static and dynamic stability of a rock slope in Manali area of Himachal Pradesh in India.

Most of these numerical codes consider the strength of rock mass as linear and generally require linear Mohr-Coulomb parameter c (cohesion) and ϕ (angle of internal friction) of slope material as strength parameters. However, the strength behaviour of rock mass is non-linear (Hobbs 1966; Ladanyi 1974; Barton 1976; lade 1977; Agar et al. 1985; Santarell 1987; Hoek and Brown 1997; Hoek et al. 2002; Baker 2004; Singh and Singh 2012)

The researchers most widely used Hoek and Brown hyperbolic criteria for stability evaluation of slopes. The researcher used equivalent Mohr-Coulomb parameters obtained by linearizing the plots of major and minor principal stress (Collins et al. 1987; yang et al. 2004; Hammah et al. 2004; Yang and Zou 2006; Li et al. 2008, 2009, 2012; Latha and Garga 2010).

Singh and Singh (2012) proposed a new criterion for jointed rock mass known as Modified Mohr Coulomb criterion. The criterion takes into account the critical state concept proposed by Barton (1976).

The authors of this article evaluated the stability of a dry rock slope on the right bank of Alaknanda river in Uttrakhand in India using limit equilibrium and numerical analysis (FEM and FDM) using FLAC 2D (Itasca Consulting Group), PHASE^2 (Rocscience) considering strength parameters of slope based on Modified Mohr-Coulomb method (Singh and Singh 2012).

2. MODIFIED MOHR-COULOMB CRITERION

Engineers are often require to predict the strength of the rock mass for design and analysis. Based on critical state concept proposed by Barton (1976), Singh and Singh (2012) proposed a new non-linear criterion for rock mass known as Modified Mohr-Coulomb criterion for jointed rock mass (Eq. 1).

$$(\sigma_1\text{-}\sigma_3) = \sigma_{cj} + \frac{2\sin\phi_{jo}}{1-\sin\phi_{jo}}\sigma_3 - \frac{1}{\sigma_{ci}}\frac{\sin\phi_{jo}}{1-\sin\phi_{jo}}\sigma_3^{\,2} \quad (\text{for } 0 \le \sigma_3 \le \sigma_{ci}) \tag{1}$$

Where, parameters σ_{ci} and σ_{cj} are uniaxial compressive strength of intact and jointed rock. ϕ_{jo} is angle of internal friction of jointed rock mass corresponding to $\sigma_3 = 0$ and is estimated from the following relationship.

$$\sin\phi_{jo} = \frac{(1-\text{SRF})+\frac{\sin\phi_{io}}{1-\sin\phi_{io}}}{(2-\text{SRF})+\frac{\sin\phi_{io}}{1-\sin\phi_{io}}} \tag{2}$$

ϕ_{io} is angle of internal friction of intact rock corresponding to $\sigma_3 = 0$ i.e. obtained from uniaxial compressive strength test and SRF is strength reduction factor obtained by Eq. 3.

$$\text{SRF} = \frac{\sigma_{cj}}{\sigma_{ci}} \tag{3}$$

3. DESCRIPTION OF CASE STUDY

3.1 Slope Section

The sections of the slope analyzed lie on right bank of the river Alaknanda. Figure 1 shows the profile of the slope sections selected for this study. The slope section is 270 m high and 360 m long and dipping towards 105°N.

3.2 Lithology

The slope section falls on rocks of Gulabkoti formation. This formation consists of quartzites of various types, dolostones with intercalations of magnesite. The quartzite is friable in nature at many places. The rocks exposed in the study area belongs to this formation. This formation has a thrusted contact on the north with Central Crystallines with the main central thrust (MCT) marking the contact.

The quartzites of Gulabkoti formation encountered close to study area are white to dirty in colour. The foliation is well developed and it dips at moderate to steep angles towards upstream and towards the right bank of river Alaknanda. Consequently, the rocks on the left bank of river had undergone sliding movements in geologic past leading to accumulation of thick debris at number of places. The human habitation is concentrated on this debris on the left bank. The right bank slope is steep to very steep with near vertical slopes at places. Further northeast, these rocks are overlain by dolomitic limestones containing pockets of magnesite.

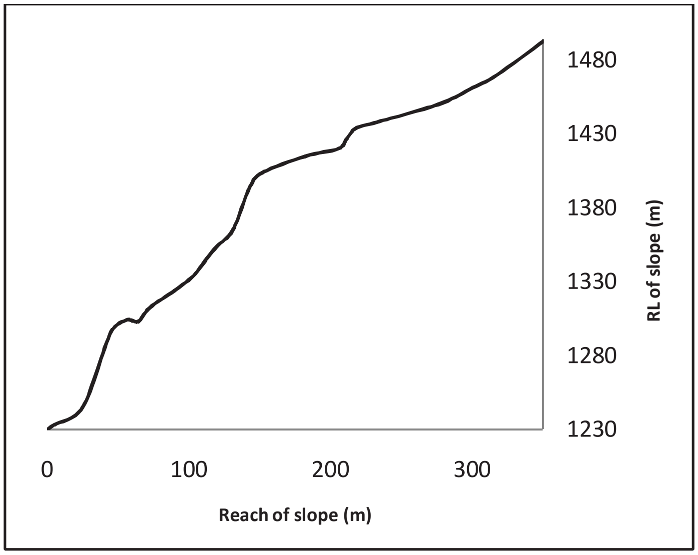

Fig. 1 - Profile of the slope section

The rocks of Gulabkoti formation are in juxtaposition with rocks of Helong formation of Central crystalline, the MCT marking the contact between the two formations. The MCT is seen to be present along a small stream joining Alaknanda river on the left bank. The Helong formation mainly consisting of quartzite, schist, augen gneisses and other high grade metamorphic rocks which are encountered further upstream of the river course. At locations of the slope sections, the structural discontinuities were observed in the field. These discontinuities were plotted on stereonet (Fig. 2) to get the preferred orientations of discontinuities. The attitudes of the discontinuities were obtained on the basis of analysis and are presented in Table 1.

Table 1- General discontinuity attitude for slope section

Sl	Joint number	Joint detail	Dip/Dip direction
1	FJ	Foliation joint	35°/N030°
2	J2	Cross joint	60°/N135°
3	J3	Vertical joint	90°/N220°

3.3 Shear Strength Parameters of Rock Mass

Field tests are the most reliable test for assessing shear strength parameters of geological media. These tests are however very expensive, time consuming and not feasible sometimes. In absence of field tests, classification approaches are used in association with field observations. Field observations on a nearby project site carried out by Singh et al. (2012) were used to assess the rock mass properties. The physical, mechanical and elastic properties of the rock were determined from laboratory testing on intact rock samples collected from the field.

Generally in slope stability studies, deterministic approach is used to determine the factor of safety. However it is recognized that the parameters governing shear strength of rock mass have large degree of uncertainty associated with them. To account for uncertainties involved, probabilistic approach was used to arrive at representative value of shear strength parameters.

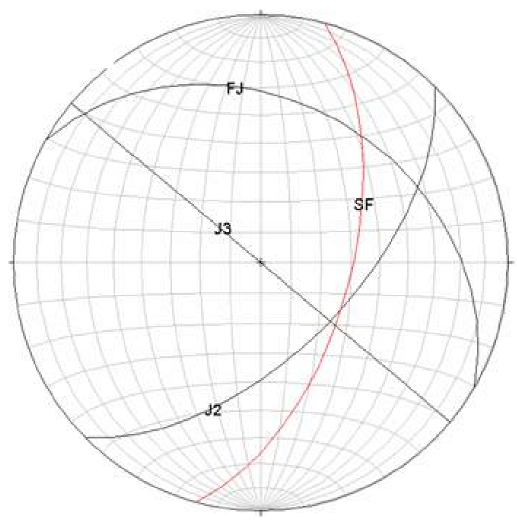

Fig. 2 - Stereo plot for major joint planes at slope section

Uniaxial compressive strength (UCS) tests were conducted on rock specimens to compute the UCS or σ_{ci} of intact rock. The results show a wide scatter in values. In order to have a single representative value of UCS of intact rock and to have a level of confidence in it, probabilistic analysis was done. Weibull distribution was fitted on the results of UCS of intact rocks and value of σ_{ci} corresponding to 50% of probability is taken as the representative value. Figure 3 shows the weibull distribution fitted on the UCS value of intact rock. The representative value of σ_{ci} is 54.86MPa.

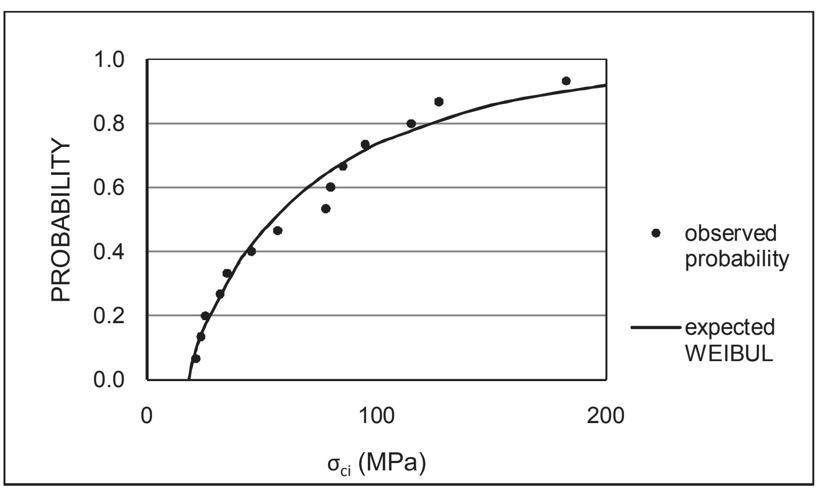

Fig. 3 - Weibull distribution curve for intact rock strength

The assessment of rock mass strength was done using the empirical relations suggested by Yudhbir & Prinzl (1983), Serafim and Pereira (1983), Ramamurthy et al. (1985), Trueman (1988), Mehrotra (1992), Kalamaras and Bieniawski (1993), Ramamurthy (1996), Sheorey (1997), Hoek and Brown (1997), Aydan and Dalgic (1998), Asef et al. (2000), Zhang (2009). Ramamurty (1993) and Hoek and Deiderich (2006) have given empirical relations for computation of elastic modulus of rock mass. Rock mass strength had been obtained from rock mass modulus using empirical equation (Eq. 4) given by Singh and Rao (2005).

$$\sigma_{cj} = \sigma_{ci} * (E_j/E_i)^{0.63} \tag{4}$$

Where E_j and E_i are elastic modulus of rock mass and intact rock respectively.

These relations required uniaxial compressive strength of intact rock and RMR or GSI or RQD value of the rock mass. Singh et al. (2012) on the basis of study of drill cores from nearby project site suggested a suitable value of RMR equal to 30 and RQD value of 27. Corresponding value of GSI is 25. This value had been used for our slope section also.

The rock mass strength was found out using the representative value of UCS of intact rock, RMR and RQD of rock mass using various empirical criteria named above. A Weibull distribution is fitted on the values of $\sigma_{c\text{-}mass}$ computed. Figure 4 shows the Weibull distribution curve fitted on the values of $\sigma_{c\text{-}mass}$ computed using empirical approach. The representative value of rock mass strength is taken equal to the value $\sigma_{c\text{-}mass}$ corresponding to 50% probability of occurrence of weibull distribution. The corresponding value of $\sigma_{c\text{-}mass}$ is 6.55 MPa.

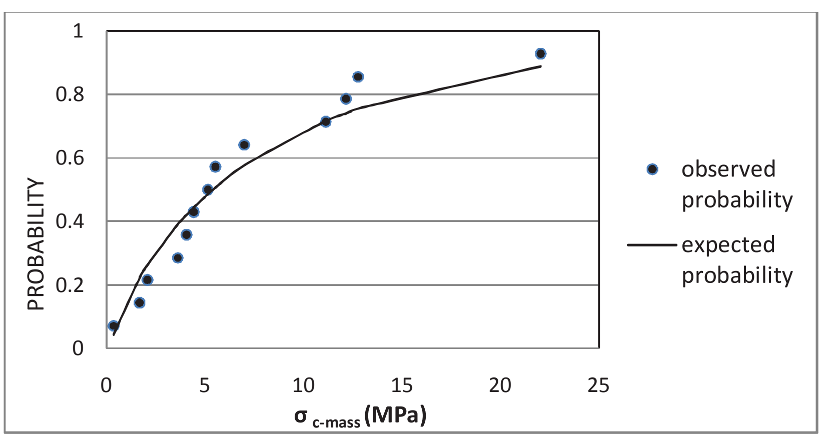

Fig. 4 - Weibull distribution on average $\sigma_{c\text{-}mass}$

Modified Mohr-Coulomb criterion for jointed rock mass require three parameters, σ_{ci}, $\sigma_{c\text{-}mass}$ and ϕ_{jo} i.e. UCS of intact rock, jointed rock mass and angle of internal friction of jointed rock mass respectively.

The value of σ_{ci} and $\sigma_{c\text{-}mass}$ is taken as representative value corresponding to 50% probability of Weibull distribution for intact and rock mass strength. The value of ϕ_{jo} can be calculated using Eqs. 2 and 3. The value of SRF comes to be 0.119. The difficulty lies in the calculation of ϕ_{io} which required triaxial data of intact rock.

Singh and Singh (2005) proposed a simple parabolic criterion for dry intact rock using the concept of critical state mechanics. Depending only on the UCS value the criterion can predict major principal stress σ_1 at a given minor principal stress σ_3 (Eqs. 5 and 6). Using this criterion major principal stress σ_1 is computed for $0 \le \sigma_3 \le \sigma_{ci}/4$. On plotting all these points in the $\sigma_1 - \sigma_3$ plane, the friction angle for the intact rock comes out to be 36.6°.

$$(\sigma_1 - \sigma_3) = A(\sigma_3)^2 - 2A\sigma_{ci}\sigma_3 + \sigma_{ci} \quad \text{For } 0 \le \sigma_3 \le \sigma_{ci} \tag{5}$$

$$A \approx -3.97(\sigma_{ci})^{-1.10} \tag{6}$$

According to Hoek et al. (2002) the value of equivalent cohesion and friction angle depends on the value of maximum confining stress in real field condition, i.e. σ_{3max}. Hoek et al. (2002) given a relationship to calculate the maximum confining stress (σ_{3max}) depending on the height of slope, uniaxial compressive strength of rock mass and unit weight of rock mass as shown in Eq. 7.

$$\frac{\sigma_{3\max}}{\sigma_{c-mass}} = 0.72\left(\frac{\sigma_{c-mass}}{\gamma H}\right)^{-0.91} \tag{7}$$

The value of $\sigma_{3\max}$ for the present case comes out to be 2.92 MPa. A negligible tensile strength (σ_t) of 0.1MPa has been assumed for the rock mass.

Since the input strength parameters required for the numerical analysis are in term of Mohr-Coulomb strength parameters, it is necessary to determine the equivalent angle of internal friction and cohesive strength for the rock mass. This is done by fitting a linear relationship to the curve generated by solving Eq. 1 for a range of minor principal stress (σ_3) values defined by $\sigma_t < \sigma_3 < \sigma_{3\max}$ as illustrated in Fig. 5.

Applying the above methodology, values of representative parameters are c_{mass}=1.42MPa and ϕ_{mass} = 43.87°. These parameters were used in the numerical analysis. Using Eqs. 5 and 8 (Singh and Singh, 2012) the dilation angle comes out to be 3.92°.

$$\phi_d = (\phi_{jo}-\phi_{io})/2 \tag{8}$$

4. NUMERICAL MODELLING

There has been considerable debate concerning the suitability of modelling jointed rock mass as a continuum. Brown (1993) suggested that when the spacing of the rock mass discontinuities is small in comparison to the scale of the problem under consideration, a continuum approach may become appropriate. *Robertson (1988)* stated that *when RMR > 40, the slope stability is governed both by orientation and shear strength of discontinuities whereas for RMR < 30 the failure develop in the rock mass similar to soil.* In the case of the selected slope, average discontinuity spacing in the deforming zone was observed to be in the range of 5 to 15 cm. For about 270 m height of the slope, this average rock block size was considered analogous to fine grained sand (grain size of 0.250 mm) in a 5.5 m high embankment. Also the RMR value for our case is 30. Consequently, modelling of the selected rock slope as a continuum deemed to be reasonable.

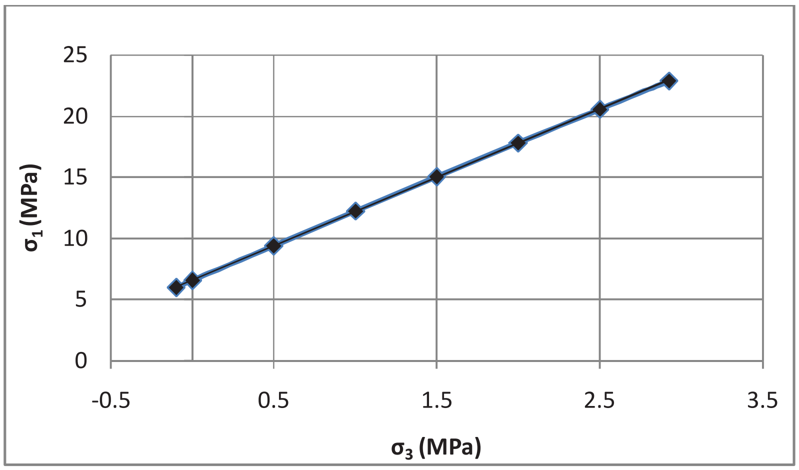

Fig. 5 - Major and minor principal stresses for modified Mohr-Coulomb criterion

The selected rock slope was modeled as a continuum material using a two dimensional, plane strain approach, based on a Mohr-Coulomb elastic-plastic formulation that is dilation upon yield. The numerical analyses were carried out using a finite difference formulation provided in the FLAC-2D version 4.0 computer code, and finite element formulation provided in the PHASE^2 version 8.008. Shear strength reduction or SSR method is used to find out the factor of safety of slopes. Table 2 presents the rock mass properties used in the analysis. Roller boundary conditions are assumed along the lateral sides of the model such that no displacement is allowed in the lateral direction. At the base of the numerical model, the boundary is fixed such that no movement is allowed in either direction.

4.1 Definition of Factor of Safety

Method for calculation of factor of safety using numerical methods differs from the conventional limit equilibrium approach. The factor of safety of a slope defined here is the factor by which the original shear strength parameter is divided in order to bring the slope to the verge of failure. For Mohr-coulomb material model the factored shear strength parameters c_f and ϕ_f are calculated as,

$$c_f = \frac{c}{SRF} \tag{9}$$

$$\phi_f = \tan^{-1}\left(\frac{\tan\phi}{SRF}\right) \tag{10}$$

Where, SRF is the "strength reduction factor (SRF)". It should be kept in mind that the SRF term is also used with Modified Mohr-Coulomb criterion but these two are different thing as defined respectively. This method is referred to as the "Shear Strength Reduction (SSR) technique" (Matsui and San 1992). In the finite element formulation the same factor is always used for both the terms. To find the true factor of safety, both FLAC and PHASE^2 performs a systematic search for the value of SRF starting from SRF=1 that will just cause the slope to fail. The final value obtained in the process is the FOS or SRF.

Table 2 - Rock mass properties at slope section

Property	Details
Rock type	Quartzite
Unit weight (N/m^3)	25000
Elastic modulus (Pa)	1.60 x10^9
Poisson's ratio	0.29
Bulk modulus (Pa)	1.27 x10^9
Shear modulus(Pa)	0.626 x10^9
Cohesion (Pa)	1.4715 x10^6
Tension (pa)	0
Angle of internal friction (degree)	43.87
Dilation angle (degree)	3.92

4.2 Finite Difference Modelling

Figure 6 shows the finite difference grid developed for the right bank section. The factor of safety is calculated using SSR technique by FLAC and it comes to be 3.21. Figures 7 to 9 show the maximum shear strain contour and factor of safety at limiting condition, plasticity indicators for limiting condition and displacement vectors for limiting condition. The nature of failure mechanism comes to be circular with a tendency of generation of tension crack in this case. In the Fig. 8 there is clear signature of tension failure on the upper part of the slope.

4.3 Finite Element Modelling

The slope section has been modelled by finite element method. The slope section was discretized with average element size of 0.5m. The boundary condition is same as in case of finite difference approach i.e. bottom of slope is restrained in both horizontal and vertical direction and side of slope is restrained only for horizontal displacement. Uniform meshing with 6 noded triangular elements is used. Total number of elements and nodes used is 171315 and 343982 respectively. The Factor of safety is calculated using SSR technique by PHASE^2 software.

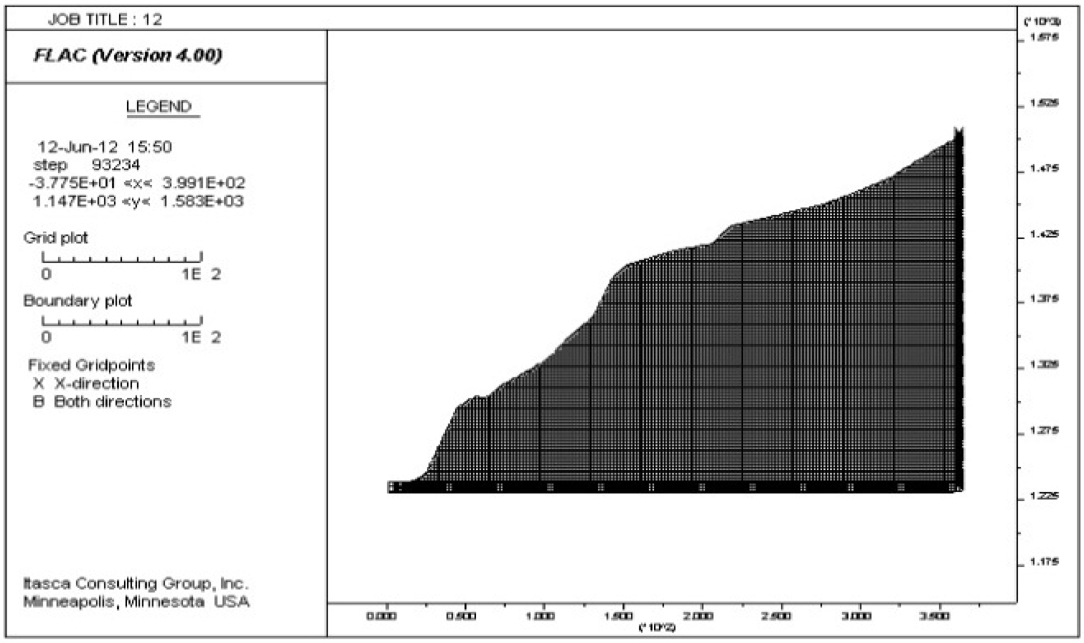

Fig. 6 - Finite difference grid for right bank slope section

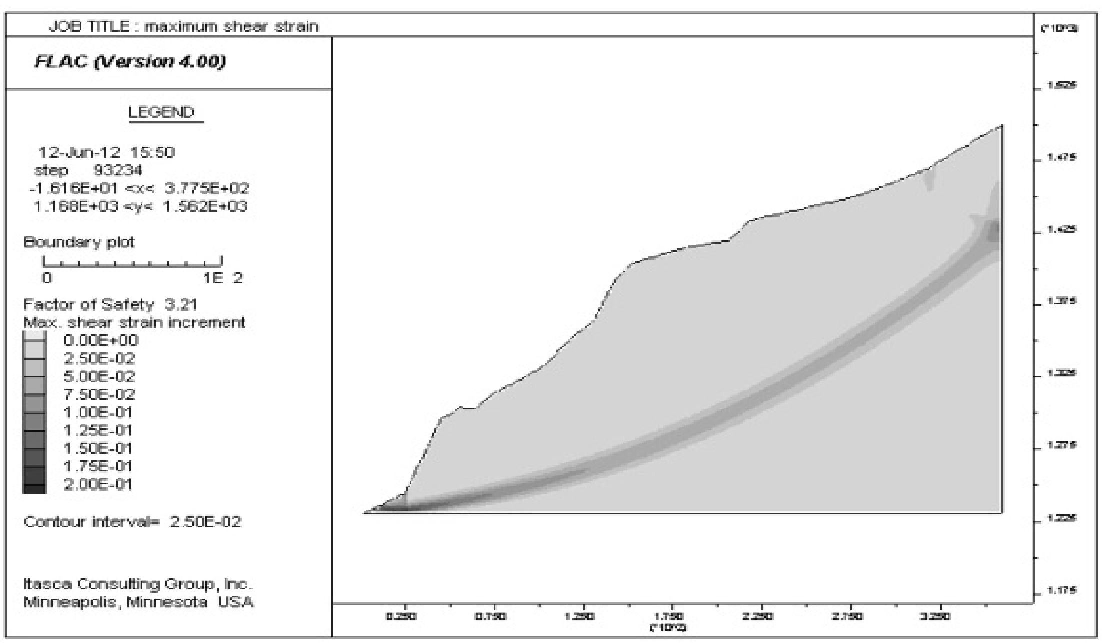

Fig. 7 - Maximum strain and factor of safety for right bank slope section

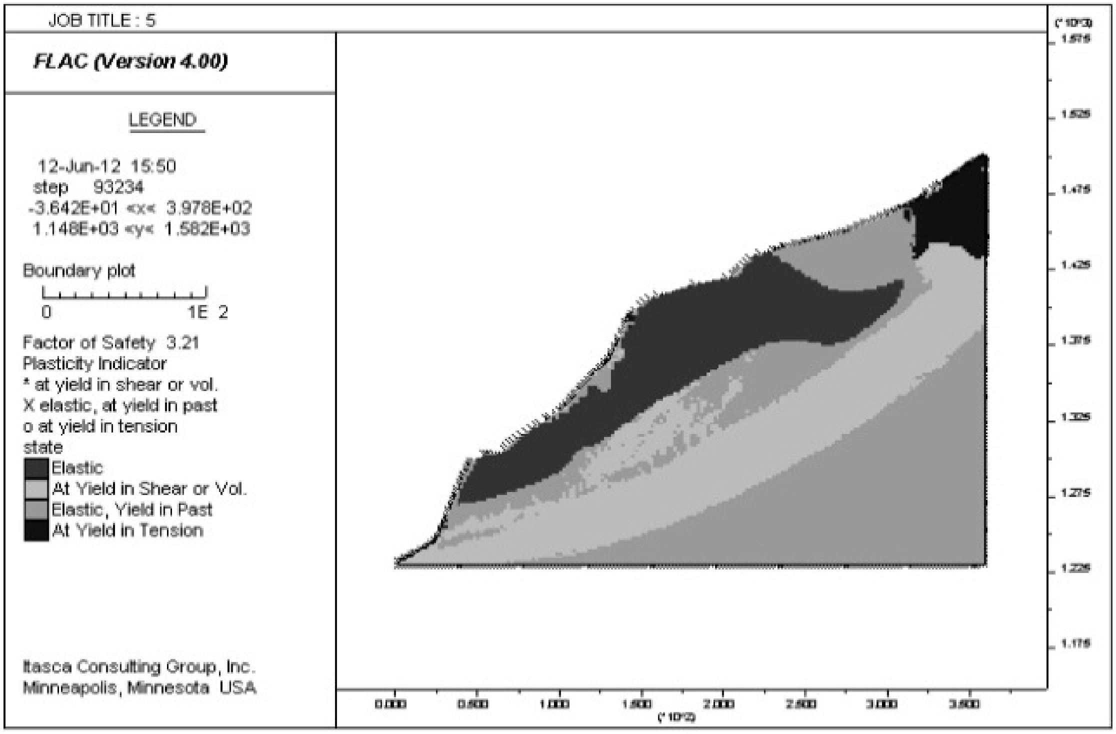

Fig. 8 - Plasticity indicator and state for right bank slope section

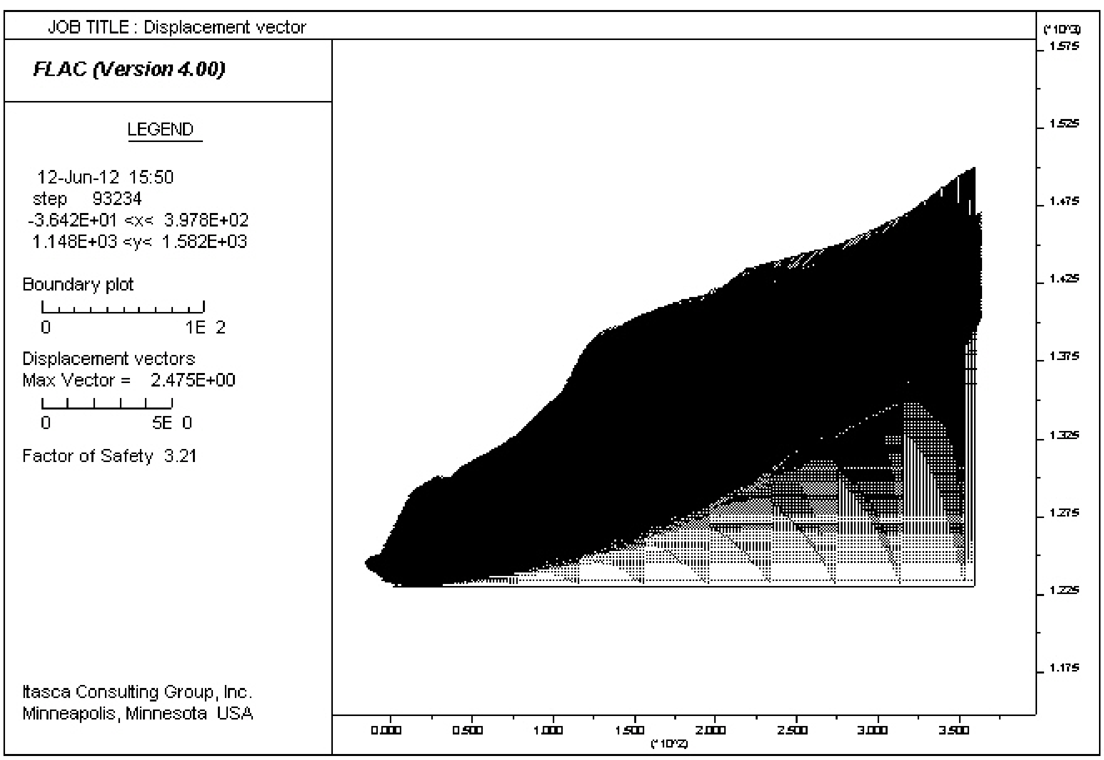

Fig. 9 - Displacement vector for right bank slope section

The Factor of safety comes to be 3.16 in this case. Figure 10 shows the finite element grid for the slope section. Figure 11 shows the maximum shear strain and factor of safety. The failure criterion used in this study is the non-convergence of the solution (Graffiths and Lane, 1999). This is the most commonly used failure criterion. When the FEM algorithm cannot converge within a user specified maximum number of iterations, the implication is such that no stress distribution can be found that can simultaneously satisfy both the Mohr-Coulomb failure criterion and global equilibrium (Graffiths and Lane, 1999). The convergence standard is controlled by the magnitude of the tolerance of out-of-balance forces, and/or the tolerance of nodal displacements specified by the user. Slope failure and numerical non-convergence occur simultaneously, and are accompanied by a dramatic increase in the nodal displacements within the mesh (Fig. 12). At the failure a band is formed within the slope in which all elements are in the plastic state and the band would go through the slope from the toe to the top (Fig. 11). The failure mechanism tendency is same circular with of generation of tension crack.

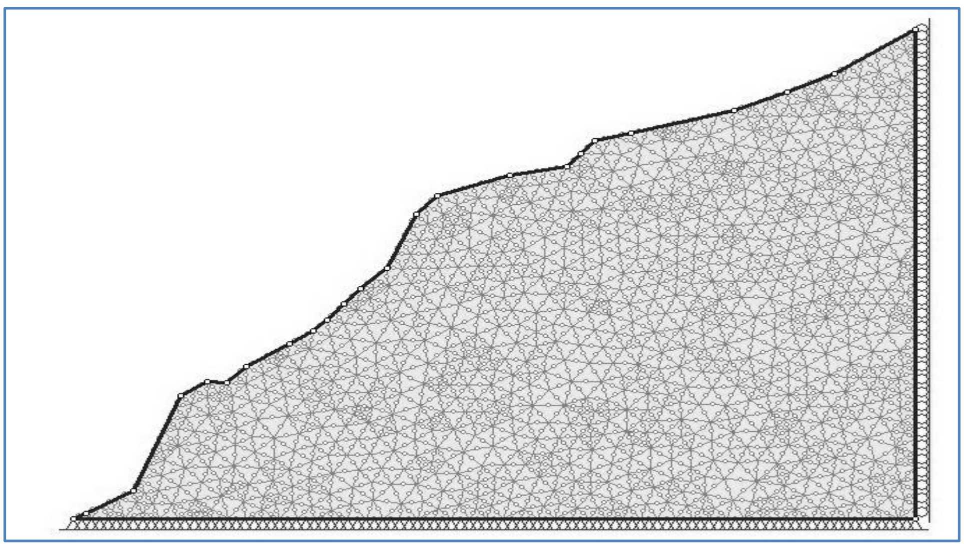

Fig. 10 - Finite element mesh for slope section

5. LIMIT EQUILIBRIUM ANALYSES

Bishop (1955) gave an effective stress analysis of which he took into account, at least partially, the effect of the forces on the vertical sides of the slices in the Swedish method. An extensive computer program was developed using C++ to find out factor of safety of the slope section using Bishop's simplified approach. The program requires the description of geometrical and strength parameters. The strength parameter i.e. c_{mass} and ϕ_{mass} used for limit equilibrium analysis is same as for numerical modelling i.e. computed from linearization of modified Mohr-Coulomb criterion.

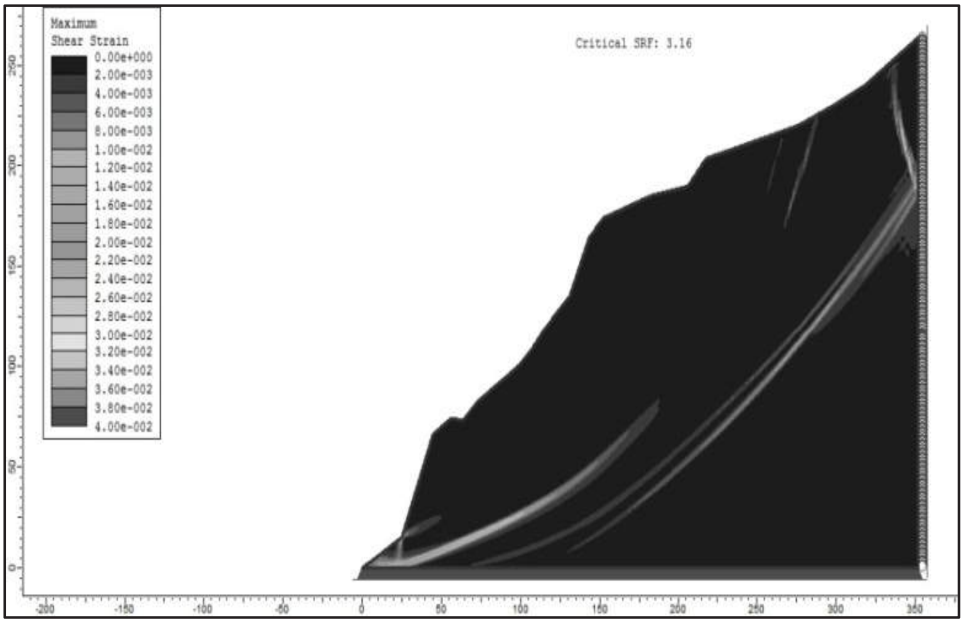

Fig. 11 - Maximum shear strain and factor of safety (SRF) for slope section

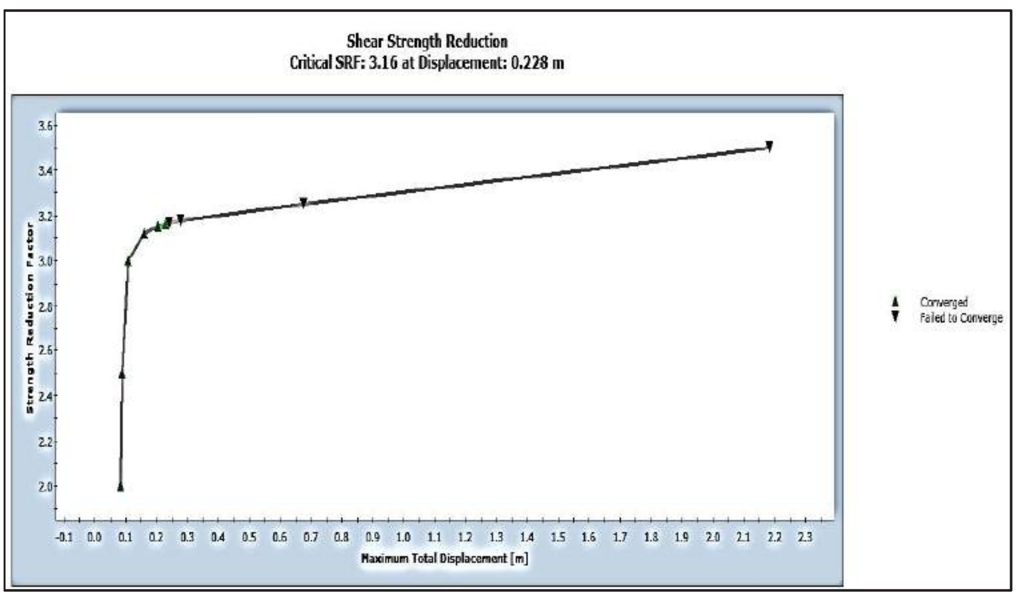

Fig. 12 - Convergence of SRF with maximum total displacement

6. RESULTS

Table 3 shows the results obtained from numerical and limit equilibrium analyses. The factor of safety computed by limit equilibrium analysis is slightly on the higher side. It is because the effect of tensile stress generated on the upper side of slope is not considered in limit equilibrium method however the numerical analysis is based on stress strain relationship so it considers the tensile stresses also.

Table 3 - Results of numerical modelling

Sl No	Approach	Software	Condition	Factor of safety	Stability	Remark
1	FDM	FLAC-2D	Dry	3.21	Stable	Circular failure mechanism, Tension crack
2	FEM	$PHASE^2$	Dry	3.16	Stable	Circular failure mechanism, Tension crack
3	Limit equilibrium method	Program for Bishop's simplified method	Dry	3.24	Stable	Circular failure

7. CONCLUDING REMARKS

Assessment of shear strength parameters required for stability analysis of rock slopes is a difficult task. The rock masses are heterogeneous in nature and lot of uncertainty is associated with them. The strength of rock mass are nonlinear in nature and most of the numerical codes used for slope stability analyses do not accept nonlinear criterion and may allow linear criteria mostly Mohr-Coulomb. To get representative values of the strength parameters, probabilistic approach incorporated with Modified Mohr-Coulomb criteria proposed by Singh and Singh (2012) has been used in the present study. The study deduces a rigorous procedure for assessment of shear strength parameters of rock mass. The procedure required the uniaxial compressive strength of intact rock which can be estimated by laboratory tests and RMR or RQD value of rock mass. Since the probabilistic approach is used, the level of confidence associated with above described procedure is high.

A slope section on the right bank of river Alaknanda was selected for stability analysis. The slope is an exposed rock mass. The strength of intact rock was found out using various tests conducted in laboratory. The RMR, GSI and RQD value for rock mass were taken equal to those of a nearby project site. Slope stability analysis was done using numerical analysis and limit equilibrium methods. The slope section is found to be stable for dry conditions when analyzed by various approaches.

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