



# Finite Element Analyses of Circular, Horse-Shoe and Interacting Circular Tunnels

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## ABSTRACT

Excavation of an underground opening in rock at depth causes redistribution of stresses in the rock and is manifested in rock deformations near the excavated surface. Thus, analysis of stress and deformations developed around an underground opening is very important in understanding the stability and support requirement of the excavation. The present study reports results of a series of elastic and elasto-plastic finite element analysis of underground openings for plane strain conditions. Hoek-Brown failure criterion has been used for elasto-plastic analysis. For the analysis of single tunnels, two shapes viz. circular and horse-shoe have been selected. In case of interacting tunnels, circular tunnels have been chosen. In each analysis, three in-situ stress conditions viz.,  $K_0 = 0.5, 1.0$  and  $1.5$  have been considered. In case of interacting tunnels, three pillar width to tunnel diameter ( $W/D$ ) ratio of  $0.3, 0.6$  and  $1.2$  have been adopted. Sequential excavation have been simulated in the analysis. The effect of shape of tunnel, tunnel interaction, sequential excavation and in-situ stress conditions have been brought out.

**Key Words:** Elasto-plasticity, Excavation, Tunnel Interaction, Underground Opening, Yield Criterion.

## I. INTRODUCTION

Underground openings have been used since ancient times for a variety of purposes. The present usage of underground openings are in hydro-electric projects, mining, railway, highway and sewage disposal networks, oil and gas storage, military installations and hazardous waste disposal etc. Underground openings are complex, costly and important structures.

The present concern is because of the availability of now less favourable sites, economics involved, ecological and environmental considerations in project execution, strategic importance and disastrous consequences in case of failure. Thus the responsibility of engineers have tremendously increased to successfully identify, understand and analyse various parameters which will have significant influence on the stability, safety and economy of any underground excavation work.

The factors which influence the stability of any underground excavation are: shape and size of opening, in-situ state of stress, induced stress field and related deformations when the opening is made, sequence of excavation in case rock behavior is in plastic region and interaction behavior of support system and surrounding rock mass and overall geological environment. The additional factors which effect the stability of multiple openings are, width of space separating the openings, depth at which the openings are being excavated and relative position of the openings.

In the present case a series of finite element analysis have been performed to study the behavior of single tunnels (circular and horse-shoe shapes) and twin circular interacting tunnels. In-situ stress states ( $K_0$ ) considered are 0.5, 1.0 and 1.5 to simulate possible field conditions. Hoek and Brown (1980) yield criterion has been used for the elasto-plastic analysis. In case of interacting tunnels three pillar width (W) to tunnel diameter (D) ratio of 0.3, 0.6 and 1.2 have been chosen. In case of single tunnels, excavation have been simulated in single stage (full face) and two stages (above springing level) in the first stage (full face) and two stages (above springing level in the first stage and remaining portion below springing level in the second stage). In case of twin circular interacting tunnels, the tunnels have been simulated to be excavated simultaneously and sequentially (right hand side tunnel first and then the left hand side tunnel) full face. In the finite element analysis carried out for plane strain conditions, eight noded isoparametric elements and  $2 \times 2$  Gauss point integration have been used. The procedure adopted to simulate excavation is that proposed by Chandrasekaran and King (1974). The elements which cover the area to be excavated are made air elements, i.e. their contribution to global stiffness matrix is reduced to almost zero. This is done by reducing the Young's modulus of the elements to be excavated to about  $10^{-6}$  th of the original value.



This presentation shows some typical results from the detailed analysis carried out of the stresses and deformations developed around the underground excavation in various cases studied.

## 2. ELASTO-VISCOPLASTICITY

Time dependent or time independent (keeping time as fictitious parameters) behaviour of soils and rocks can be analysed by using the theory of elasto-viscoplasticity.

In the present study elasto-viscoplasticity theory developed by Zienkiewicz and Corneau (1974) has been adopted and used as an artifice to obtain elasto-plastic solution. The theory is briefly presented in this section. The state of stress at a point is represented by the stress vector  $\{\sigma\}$ ,  $(\{\sigma\}^T = [\sigma_x \ \sigma_x \ \sigma_x \ \tau_{xy} \ \tau_{yz} \ \tau_{zx}])$  and total strain by the vector  $\{\varepsilon\}$ ,  $(\{\varepsilon\}^T = [\varepsilon_x \ \varepsilon_x \ \varepsilon_x \ \gamma_{xy} \ \gamma_{yz} \ \gamma_{zx}])$

The total strain at a point is the sum of plastic and viscoplastic strains, i.e.

$$[1] \quad \{\varepsilon\} = \{\varepsilon^e\} + \{\varepsilon^{vp}\}$$

where  $\{\varepsilon^e\}$  = elastic strain vectors  
 $\{\varepsilon^{vp}\}$  = viscoplastic strain vectors

The stresses are related to the total and viscoplastic strains through the relation,

$$[2] \quad \{\sigma\} = [D](\{\varepsilon\} - \{\varepsilon^{vp}\})$$

where  $[D]$  = elasticity matrix. The elements of  $[D]$  are function of Young's modulus and Poisson's ratio.

The yield function in general can be written in the form

$$[3] \quad F = F(\{\varepsilon\}, \{\varepsilon^{vp}\}) = 0$$

Equation 3 represents in general the conditions of plasticity, hardening/softening at a point. The viscoplastic strain rates are given by the flow rule

$$[4] \quad \{\varepsilon^{\text{vp}}\} = \mu \langle F \rangle \frac{\partial F}{\partial \{\sigma\}}$$

where  $\{\varepsilon^{\text{vp}}\}$  = viscoplastic strain vector,  
 $\mu$  = fluidity parameter (taken as 1 for elasto-plastic analysis),  
 $\langle F \rangle = \begin{cases} 0, & \text{if } F \leq 0 \\ F, & \text{if } F > 0 \end{cases}$

### 3. YIELD OR FAILURE CRITERION

The empirical failure criterion as proposed by Hoek and Brown (1980) can be written as

$$[5] \quad \sigma_1 = \sigma_3 + (m\sigma_c \sigma_3 + s\sigma_c^2)^{1/2}$$

where  $\sigma_1$  = major principal stress at failure,  
 $\sigma_3$  = minor principal stress applied to the sample,  
 $\sigma_c$  = uniaxial compressive strength of the intact rock material in the specimen,  
 $m$  and  $s$  = constants which depend upon the properties of the rock and upon the extent to which it has been broken before being subjected to stress  $\sigma_1$  and  $\sigma_3$

The uniaxial tensile strength  $\sigma_t$  of the specimen is given by substituting  $\sigma_1 = 0$  in Eqn. [5] and by solving the resulting quadratic equation for  $\sigma_3$ . Thus,

$$[6] \quad \sigma_t = \frac{1}{2} \sigma_c \left\{ m - (m^2 + 4s)^{1/2} \right\}$$

Thus Eqn. [5] includes both, failure criterion of rock and limited tension together. Thus a separate 'No Tension' analysis is not required.

Equation [5] can be written in the form

$$[7] \quad F = \sigma_1 - \sigma_3 - (m\sigma_c \sigma_3 + s\sigma_c^2)^{1/2} = 0$$

Thus the original rock mass is linear elastic until  $F \leq 0$  and goes to plastic state when  $F > 0$ .

For the numerical computation it is convenient to write the yield function in terms of three stress invariants,  $\sigma_m$  (mean stress),  $\bar{\sigma}$  (second invariant of deviatoric stresses) and  $\theta_0$  (Lode angle) as proposed by Nayak and Zienkiewicz (1972). In terms of three stress invariants, the yield criterion (Eqn. 7) can be written as

$$[8] \quad F = \frac{4\bar{\sigma} \cos^2 \theta_0}{\sigma_c} + m \left( \cos \theta_0 + \frac{\sin \theta_0}{\sqrt{3}} \right) \bar{\sigma} - m\sigma_m - s\sigma_c = 0$$

#### 4. CASES ANALYSED

A schematic diagram shown in Fig. 1 indicates the tunnel shapes, diameter, material properties and In-situ stress ratios chosen for the analysis.

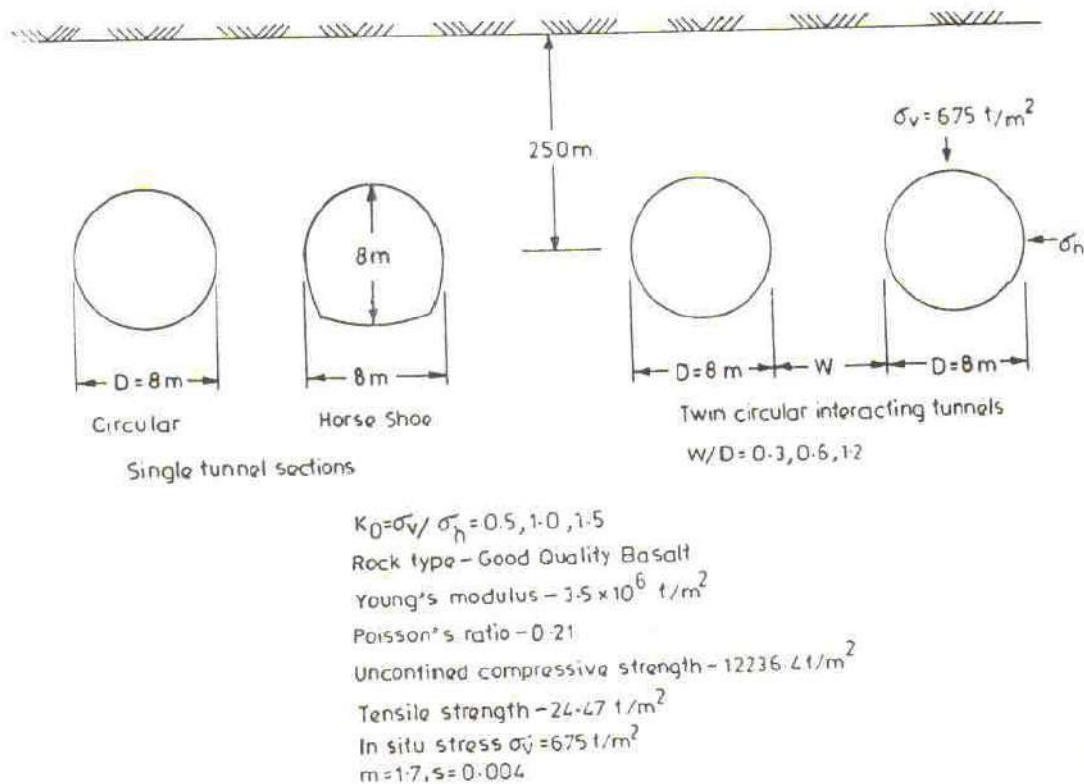


FIGURE 1 Schematic Diagram — Single and Interacting Tunnels

### 5. RESULTS AND DISCUSSIONS

In this section some typical results from the analyses of various cases carried out for single tunnels of circular and horse-shoe shapes and twin circular in-teracting tunnels are being presented.

#### 5.1 Circular Tunnel

Deformations at the surface of excavated tunnel have been obtained for  $K_0 = 1.0, 0.5$  and

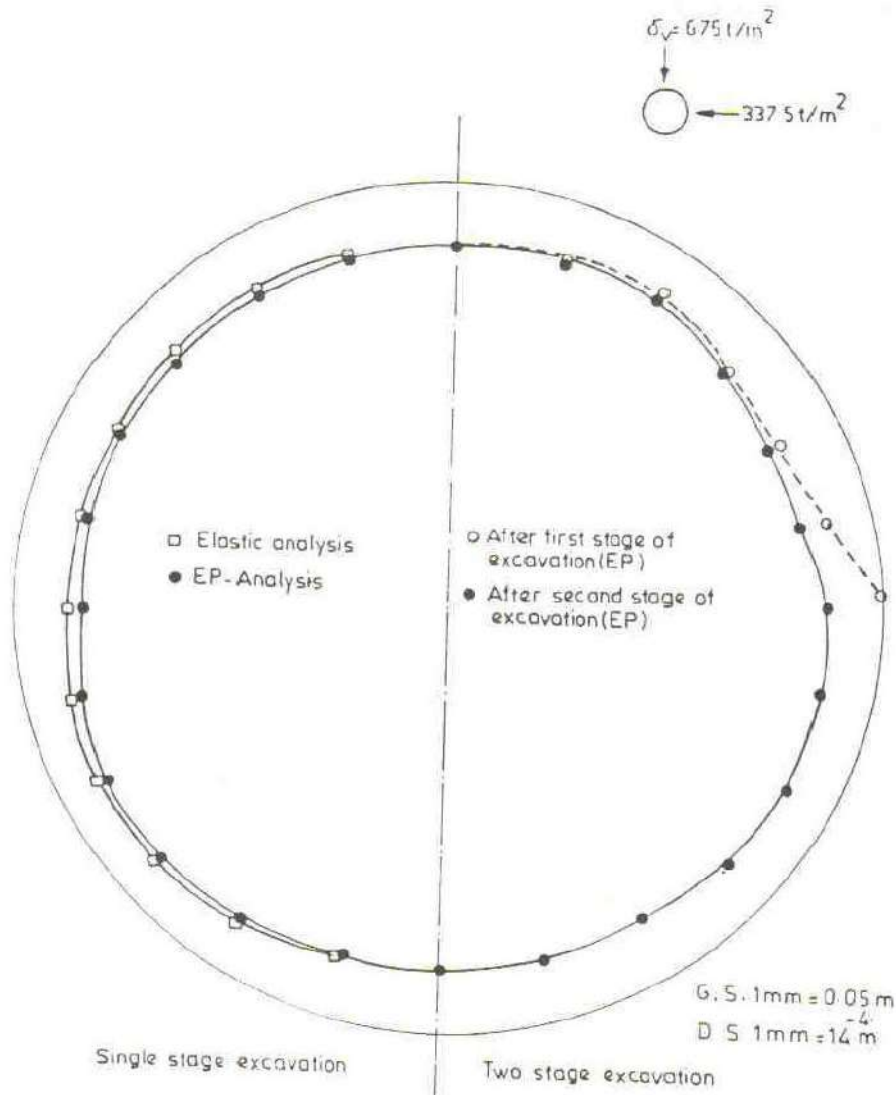


FIGURE 2 Deformed Shape of Tunnel ( $K_0 = 0.5$ )



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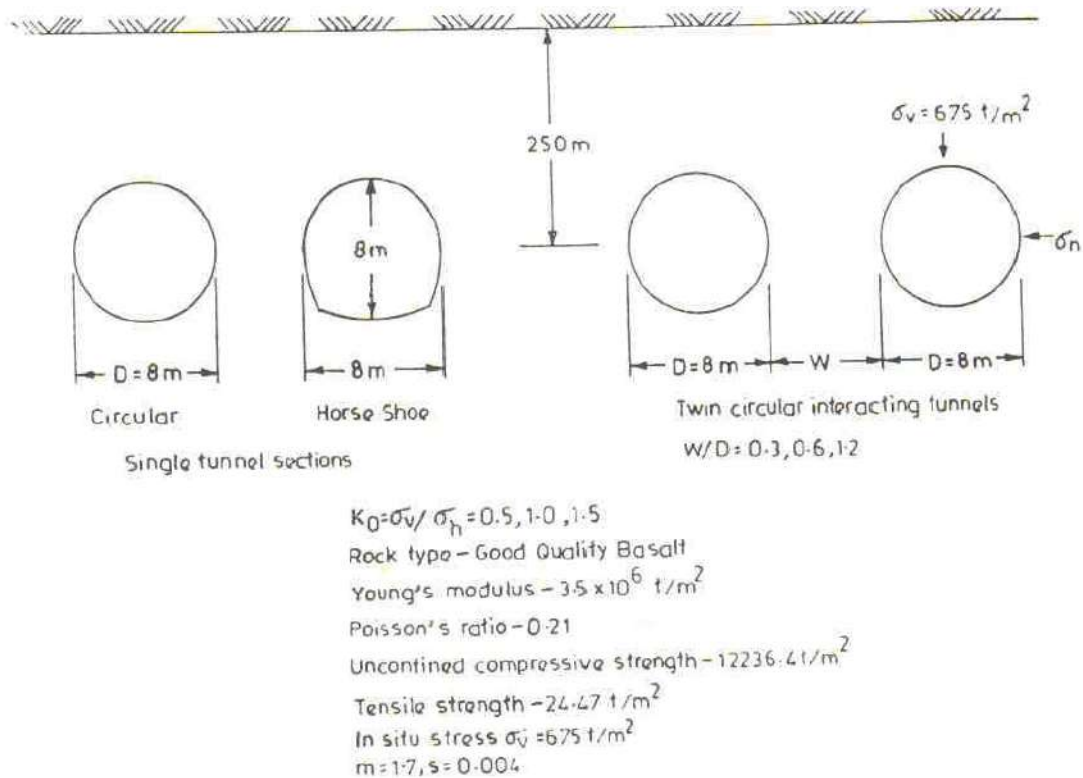


FIGURE 1 Schematic Diagram — Single and Interacting Tunnels

1.5 conditions. The deformed shape of tunnel for  $K_0 = 0.5$  and  $1.5$  are shown in Figs. 2 and 3. In these figures, on the left hand side results of single stage excavation and on the right hand side results of two stage excavation (excavation above the springing level in the first step and remaining portion in the second step) are shown.

In case of in-situ stress ratio  $K_0 = 1.0$  (figure not shown) it has been observed that for single stage excavation, the deformed shape of the tunnel is uniform and concentric to the tunnel boundary for both elastic and elasto-plastic analyses. For two stage excavation, in case of elastic analyses, the sequence of excavation has no effect as expected also but for elasto-plastic analysis the final deformed shape obtained is non-uniform. Since  $K_0 = 1.0$ , in single

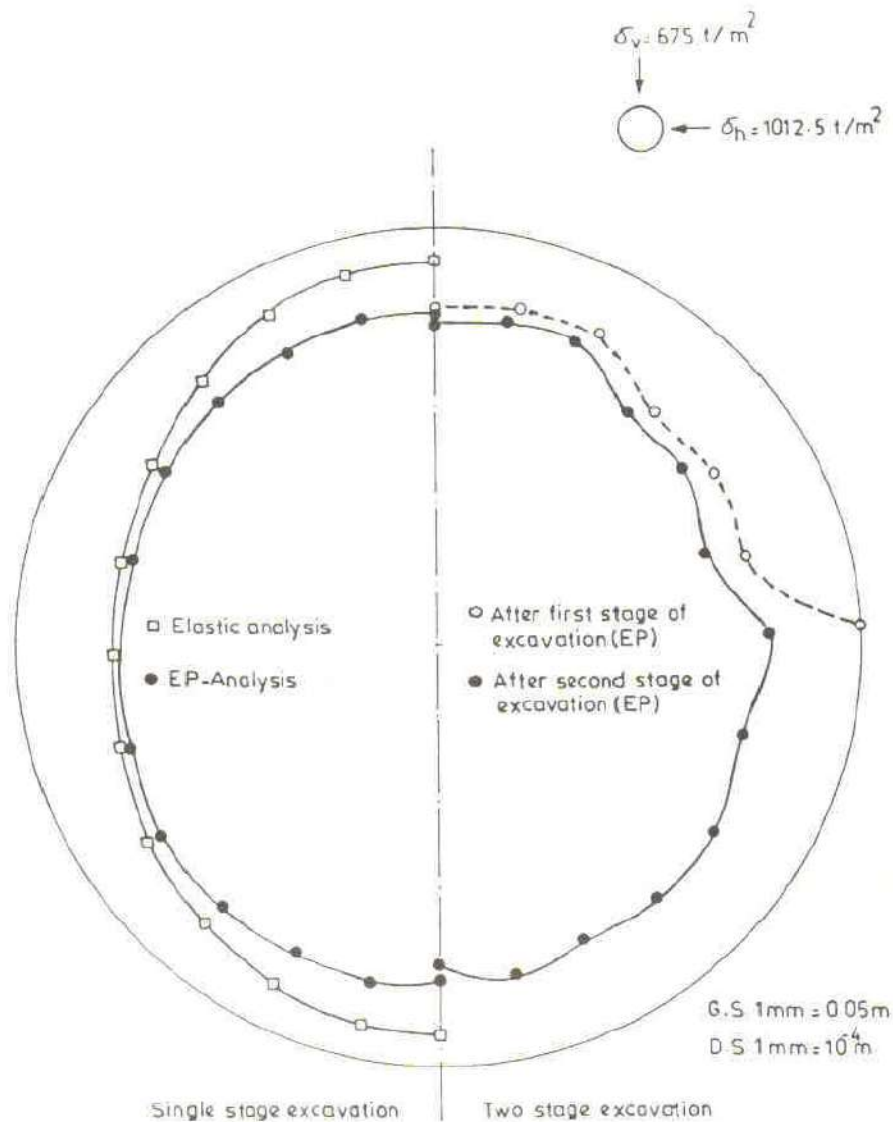


FIGURE 3 Deformed Shape of Tunnel ( $K_0 = 1.5$ )



stage excavation, the difference in displacements between elastic and elasto-plastic analysis is uniform around the periphery of the tunnel. In case of two stage excavation the final deformation around the tunnel periphery are non uniform with maximum deformation between crown and springing level.

In case of in-situ stress ratio  $K_0 = 0.5$  it has been observed that the maximum deformation takes place at the crown of the tunnel and minimum at the springing level of the tunnel in case of elastic analysis. In case of elasto-plastic analysis, the deformation near the springing increase markedly as compared to those obtained from elastic analysis. In case of elasto-plastic analyses for single stage excavation, it can be seen that near the tunnel boundary, the deformation tends to be uniform all around the tunnel whereas for two stage excavation it is non- uniform. A comparison of displacements obtained at tunnel boundary from various cases analysed indicates that the maximum displacements are obtained in case of two stage excavation elasto-plastic analysis.

For the case of in-situ ratio  $K_0 = 1.5$ , it has been observed that for elastic analysis, the maximum displacement takes place at the springing level of the tunnel and minimum at the crown of the tunnel. In case of elasto-plastic analysis, the deformation near the crown of the tunnel increase markedly as compared to those obtained from elastic analysis. The deformed shape obtained from elasto-plastic analysis for two stage excavation is markedly non- uniform in this case. A comparison of the displacements obtained from elasto-plastic analysis for two stage excavation show that is general, the displacement in the former case are higher and in the upper half of the tunnel, the difference in the displacements is more as compared to the lower half of the tunnel.

Principal stress contours for  $K_0 = 0.5$  and 1.5 conditions for two stage excavation (elasto-plastic analysis) are shown in Figs. 4 and 5 respectively. A comparison of stress contours from elastic analysis and single stage excavation elasto-plastic analysis (figure not shown) shows that major principal stresses are significantly affected by yielding of rock and multiple stage excavation and the effect is more in case of higher in-situ stress ratio.

For the single excavation, development of any tension around the tunnel is not indicated in the analysis, whereas for two stage excavation tension develops near the springing level after the first stage of excavation for the present excavation scheme and this is taken care of by Hoek-Brown yield criterion. A separate 'No Tension' analysis is not required to be done.

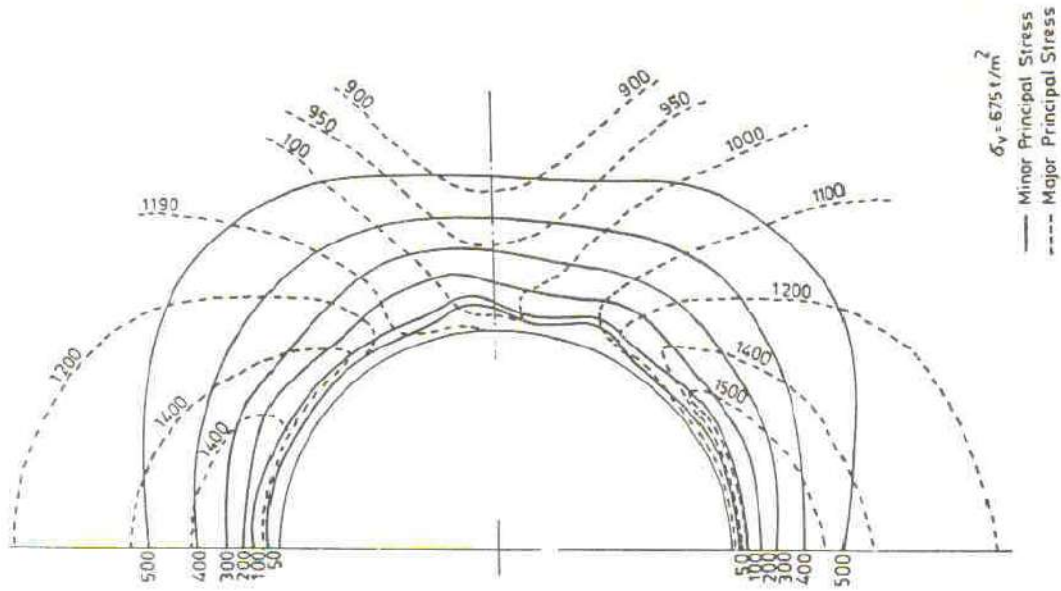


FIGURE 5 Principal Stress Contours – Two Stage Excavation – EP Analysis ( $K_o = 1.5$ )

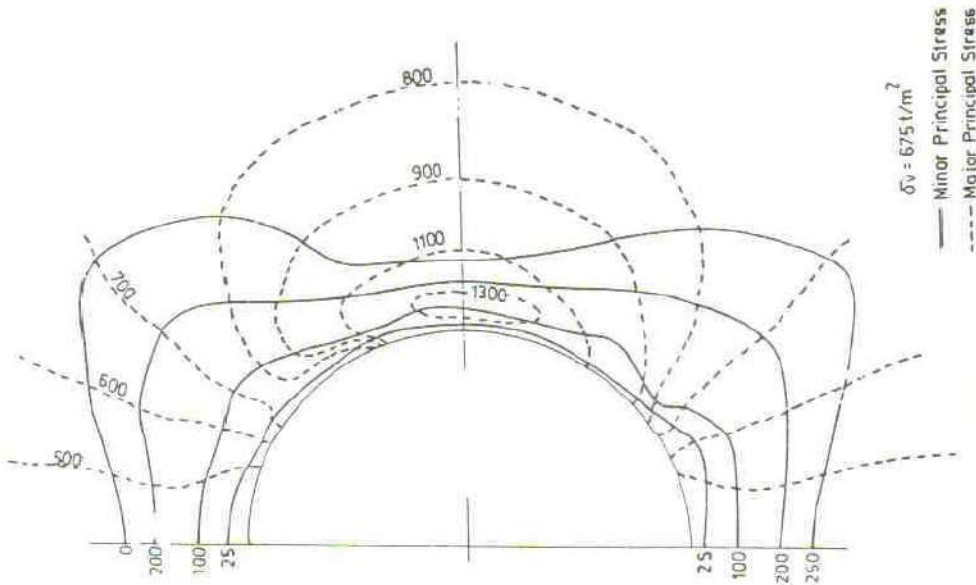


FIGURE 4 Principal Stress Contours – Two Stage Excavation – EP Analysis ( $K_o = 0.5$ )

## 5.2 Horse-shoe Tunnel

The cross-sectional area of this tunnel is approximately 4% more than the circular tunnel. The deformed shape of tunnel for the three in-situ stress conditions  $K_0 = 1.0, 0.5$  and  $1.5$  have been obtained for elastic and elasto-plastic analysis. Figs. 6 and 7 present these shapes for  $K_0 = 0.5$  and  $1.5$ . Similar to the case of circular tunnels, in these figures also on the left hand side results of single stage excavation and on the right hand side results of two stage excavation are shown.

In case of in-situ stress ratio  $K_0 = 1.0$  (figure not shown), for single stage excavation, the deformed shape for the elastic and elasto-plastic analysis are almost same as the shape of the tunnel. In case of excavation in two stages, for elastic analysis, the final deformed shape of tunnel is same as that for single stage excavation as expected. And for elasto-plastic analysis the displacements are observed to be non-uniform as compared to shape of tunnel periphery.

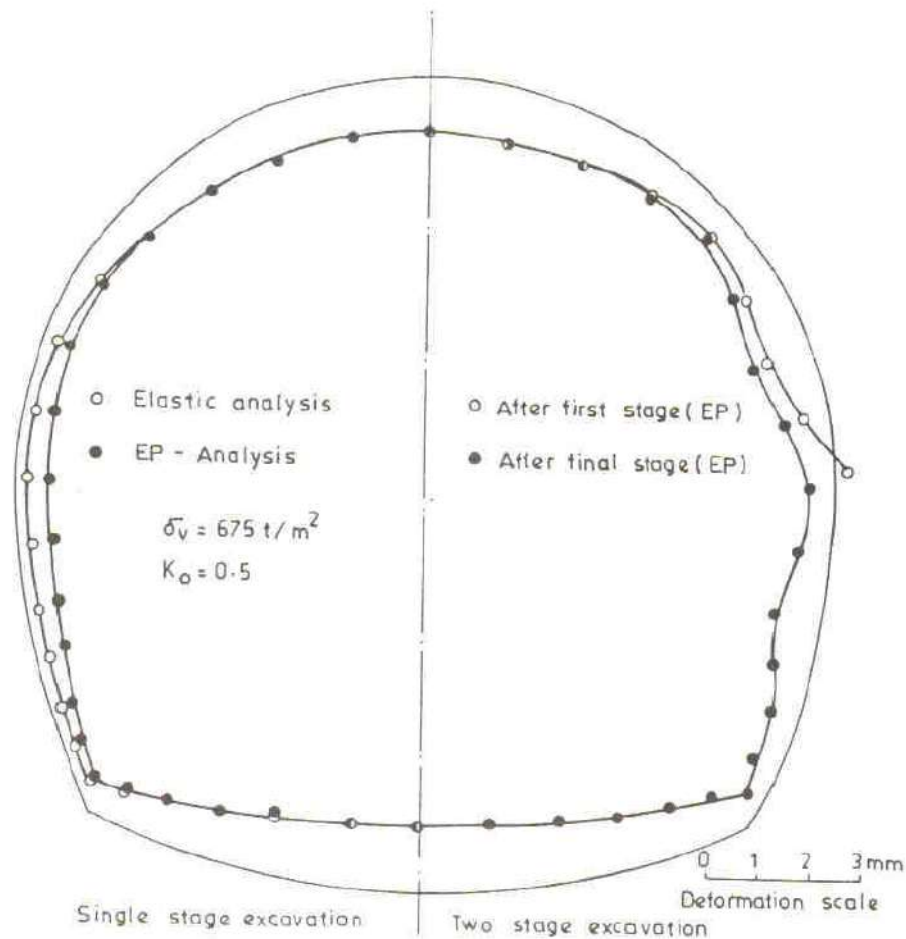


FIGURE 6 Deformed Shape of Tunnel ( $K_0 = 0.5$ )



For  $K_0 = 0.5$  condition, considering single stage excavation, the elastic deformations are higher at the crown and invert portion of the tunnel. For two stage excavation elasto-plastic deformed shape is non-uniform. As compared to single stage excavation, it is found that above the springing level, the displacements are higher in two stage excavation but around springing level they are lower in this case and then again are slightly higher than the case of single stage excavation. In the invert portion of the tunnel, the difference is almost negligible.

In case of  $K_0 = 1.5$  condition, for single stage excavation, the elastic deformations are higher near springing level of the tunnel and smaller in the crown and invert portion of the tunnel. The elasto-plastic deformation are more in the crown and invert portion of the tunnel and in the sidewall portion they are smaller. For two stage excavation, the elasto-plastic analysis indicates large deformations in the side wall of the tunnel in the second stage of excavation. The solution does not converge and comparatively large deformations and instability are indicated. Compared to two stage excavation, the single stage excavation indicates lesser

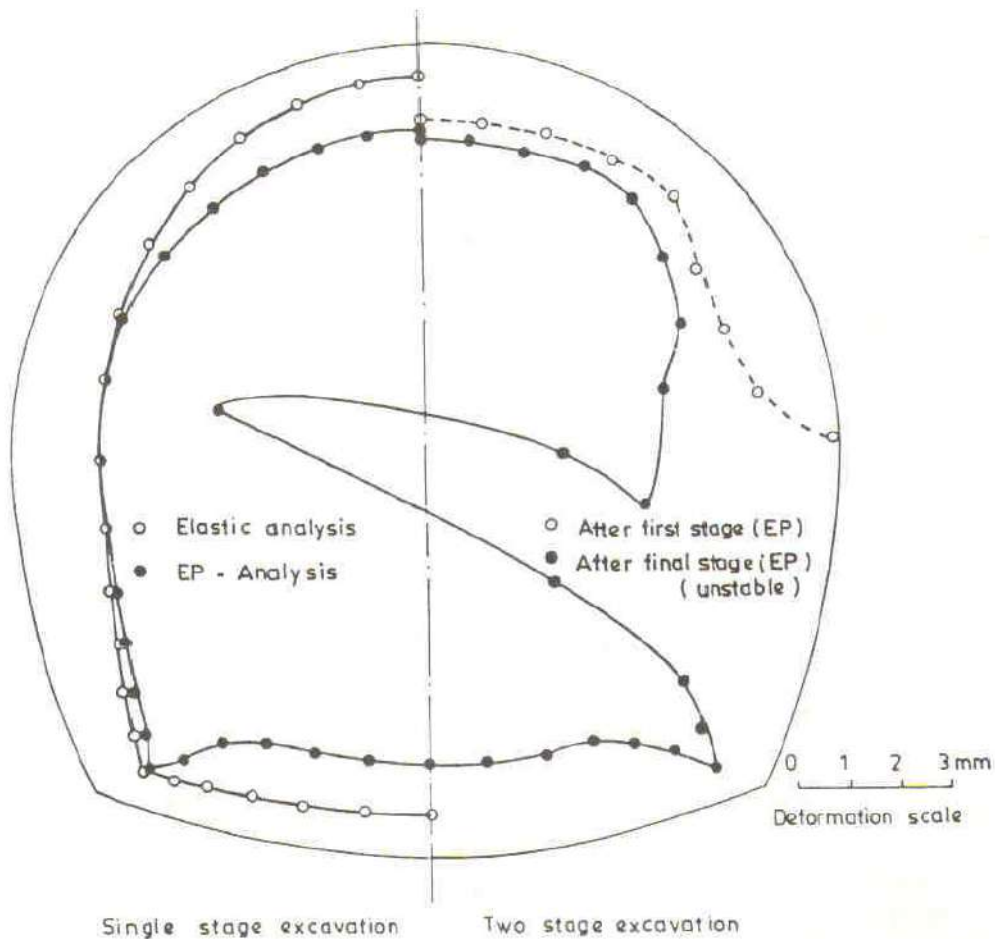


FIGURE 7 Deformed Shape of Tunnel ( $K_0 = 1.5$ )

deformations and stability for the elasto-plastic behaviour of the geologic material.

Principal stress contours for  $K_0 = 0.5$  for two stage excavation (elasto-plastic analysis) are shown in Fig. 8. The contours, in general show the stress concentration zone near the bottom corner of the tunnel. The effect of yielding of rock in this case is more pronounced in case of minor principal stress. In case of  $K_0 = 1.5$  condition, Fig. 9 shows stress contours for single stage excavation, elasto-plastic analysis. Since the elasto-plastic analysis indicates a comparative instability of side wall, in two stage excavation, hence the principal stress contours are not shown. In general the contours show stress concentration zone near the bottom corner of the tunnel. A comparison of stress contours for the elastic (not shown here) and elasto-plastic analyses shows that both the principal stresses are affected by yielding of the geologic material but in this case it is more pronounced in case of major principal stresses. The Maximum value of tensile stress is indicated for in-situ stress  $K_0 = 1.5$ . In two stage excavation scheme after the first stage excavation, it is near the springing level ( $-299.6 \text{ t/m}^2$ ). This is reduced to  $-5.7 \text{ t/m}^2$  after the first stage elasto-plastic solution is obtained. The elasto-plastic solution after the second stage excavation does not converge.

### 5.3 Comparison of circular and Horse-shoe Shaped tunnel Behavior

For both the shapes of tunnels some similarities and some differences in behavior have been observed. These are described as follows :

For the present material properties and in-situ states, if more yielding is indicated, the effect of sequence of excavation is more and conversely, if no yielding is indicated, sequence of excavation does not have any effect. Because as expected, the final displacements and stresses are independent of the sequence of excavation in elastic analysis and very much dependent on the sequence of excavation in elasto-plastic analysis.

It has been found that the sequence of excavation can result in large reversal of stresses and in the direction of movement of the points around the tunnel.

In case of elastic analysis, displacements are very much a function of in-situ stress ratio and the difference in displacements between any two points is more for stress ratios other than 1. In case of elasto-plastic analysis, the displacement distribution tends to be comparatively uniform (for  $K_0$  values not differing much from 1) and the difference in displacements between any two points is comparatively less as compared to elastic analysis.

From a comparison of yielding, displacements and stresses for the two shapes of tunnels and for the properties used, it has been observed that for in-situ stress ratio  $K_0 > 1$ , in case of circular shape of tunnel, less yielding and displacements are obtained as compared to horse-shoe shaped tunnel and for  $K_0 < 1$  the reverse trend is observed.



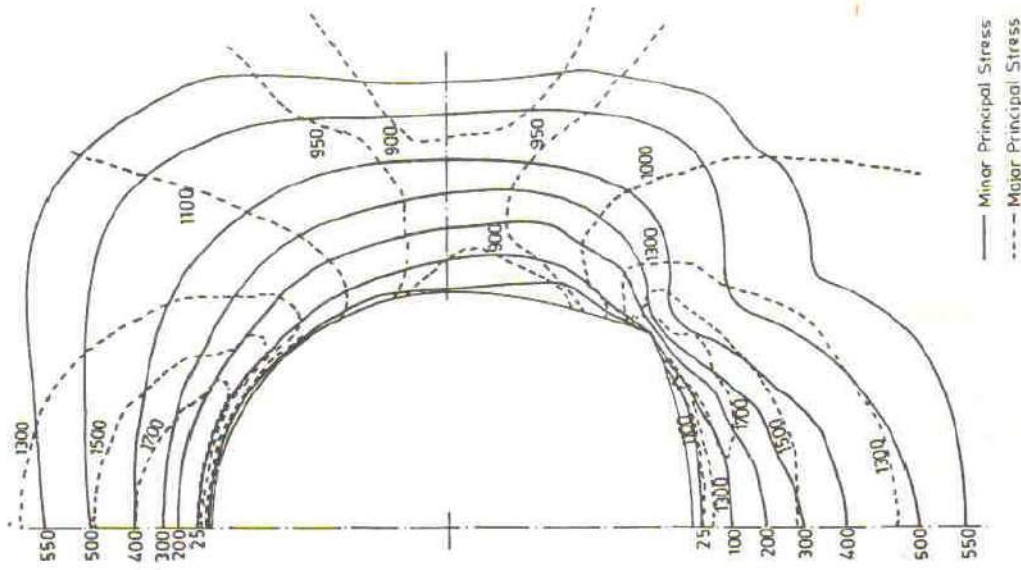


FIGURE 9 Principal Stress Contours - Two Stage Excavation - EP Analysis ( $K_o = 1.5$ )

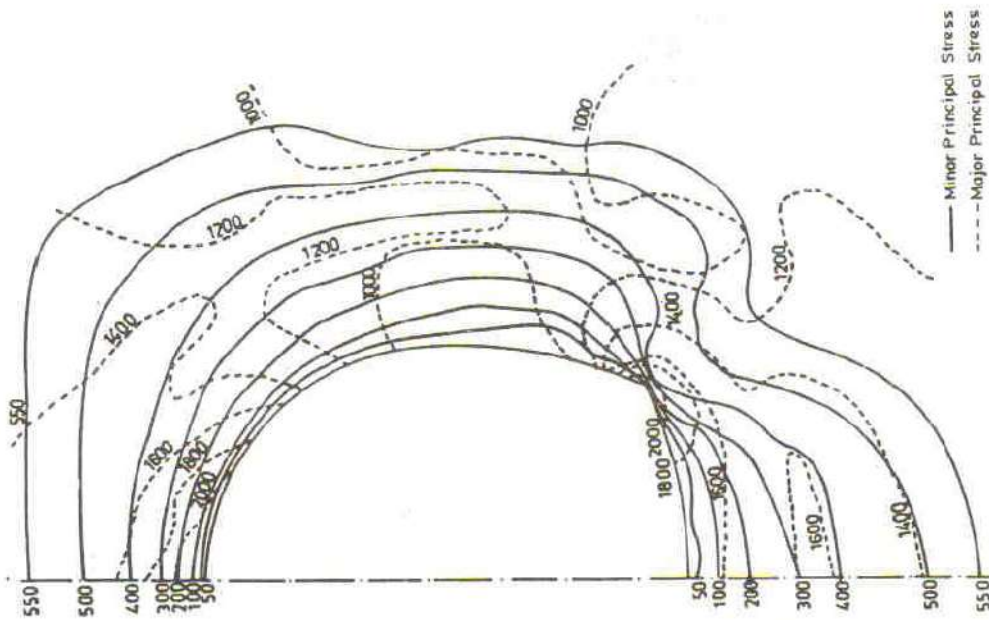


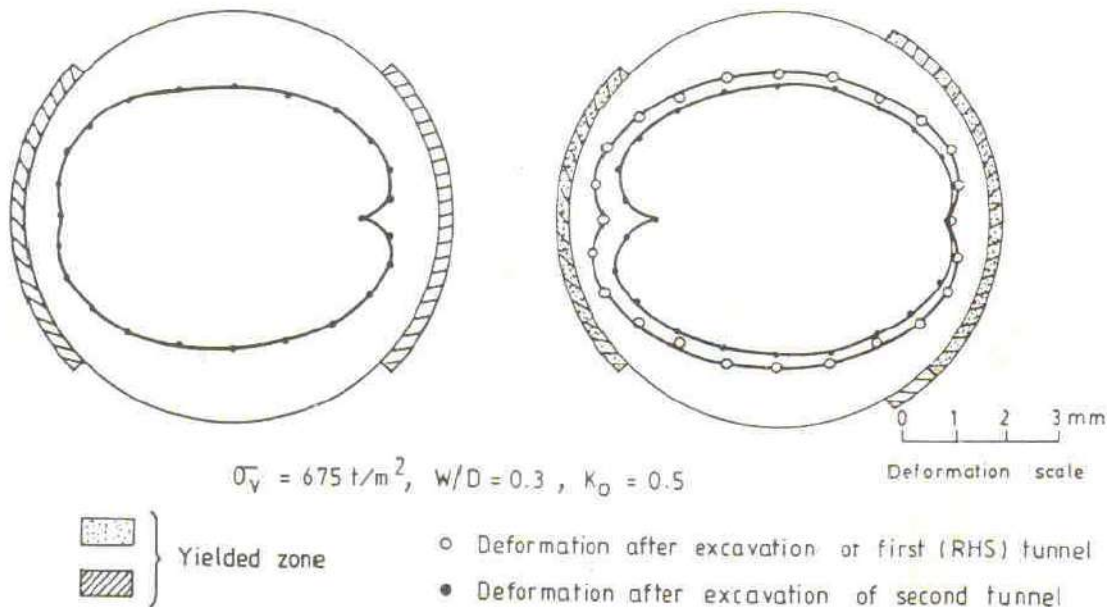
FIGURE 8 Principal Stress Contours - Two Stage Excavation - EP Analysis ( $K_o = 0.5$ )



For the two shapes of tunnels analysed, a comparison of single and two stage excavations shows that single stage excavations should be preferably carried out because in multiple stage excavation, the media is likely to be subjected to tensile stresses also, and the portions which have been excavated in previous stages are subjected to increasing stresses (because of stress redistribution) and hence larger displacements occur in the excavated portion of tunnel. Moreover, uniformity of displacements and stresses is also lost this may create difficulties in the choice and provision of support structures.

### 5.4 Interacting tunnels

From the various cases analysed, for  $W/D = 0.3$  and  $K_0 = 0.5$  and  $1.5$  the yielded zone and deformed shapes of the tunnel are shown in Figs. 10 and 11 for sequential excavation elasto-plastic analysis. It has been observed that the yield zone is higher in-situ ratio of  $1.5$  and deformed shapes are less uniform in case of lower in-situ stress ratio of  $0.5$ . In case the material behaviour is elastic (deformed shape not shown), the deformations are least in all cases at the springing level on the pillar side and amongst the various in-situ stress ratios,



these are least for lower in-situ stress ratio of  $K_0 = 0.5$ . In case of elasto-plastic analysis for all the cases, the deformations are comparatively larger at springing level on the pillar side and except at this location they tend to be concentric to the shape of the tunnel boundary. Compared to single tunnels, in case of elastic analysis, the difference in deformation is maximum for higher in-situ stress ratio of  $K_0 = 1.5$  but for elasto-plastic analysis, the difference is maximum for lower in-situ stress ratio of  $K_0 = 0.5$ . A comparison of sequential and simultaneous excavation of interacting tunnels shows that the difference in elasto-plastic and elastic analysis is maximum for lower in-situ stress ratio of  $K_0 = 0.5$  and least for  $K_0 = 1.5$ .

Major Principal stress contours for elasto-plastic analysis simultaneous excavation of tunnels  $W/D = 0.3$  and for  $K_0 = 0.5$  and  $1.5$  are shown in Figs. 12 and 13. The difference with stress contours of sequential excavation is not appreciable (hence not shown). A comparison of stress contours on the pillar and abutment (where effects of interaction are obviously small) sides, show that in the pillar portion a zone of relatively higher stress gradients exists. It is observed that the difference is appreciable for  $K_0 = 0.5$  and very marked in case of  $K_0 = 1.5$  showing considerable effect of interaction for higher in-situ stress ratio.

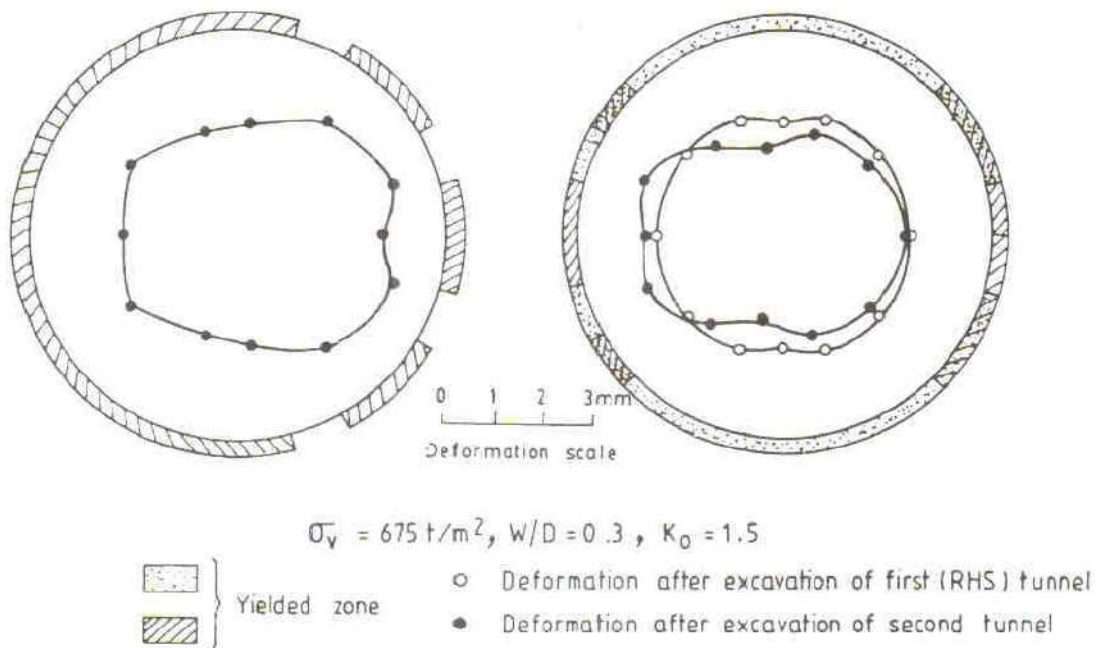


FIGURE 11 Deformed Shape of Interacting Tunnel – Sequential Excavation – EP Analysis

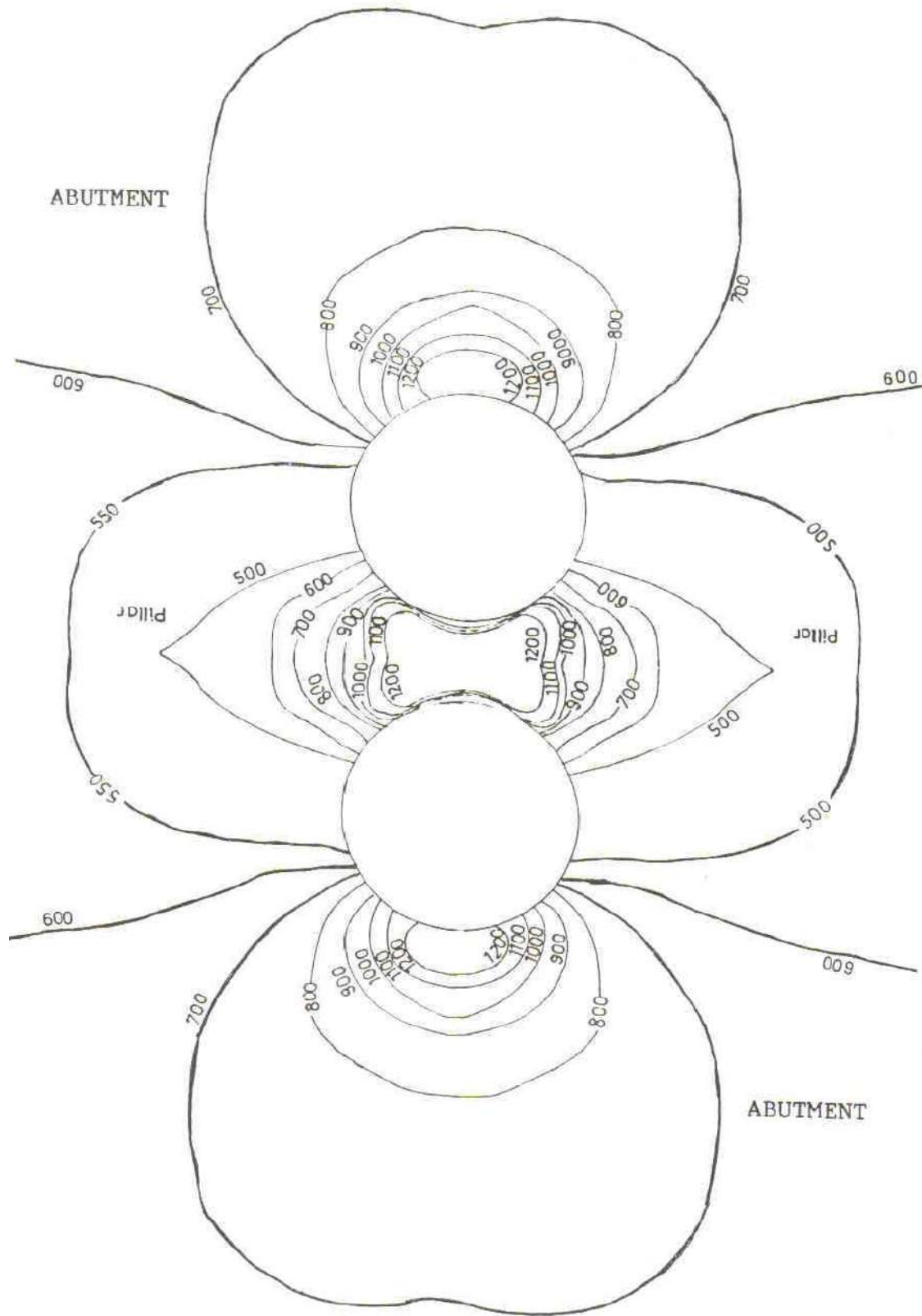


FIGURE 12 Major Principal Stress Contours ( $W/D = 0.3$ ,  $K_0 = 0.5$ ) – EP Analysis



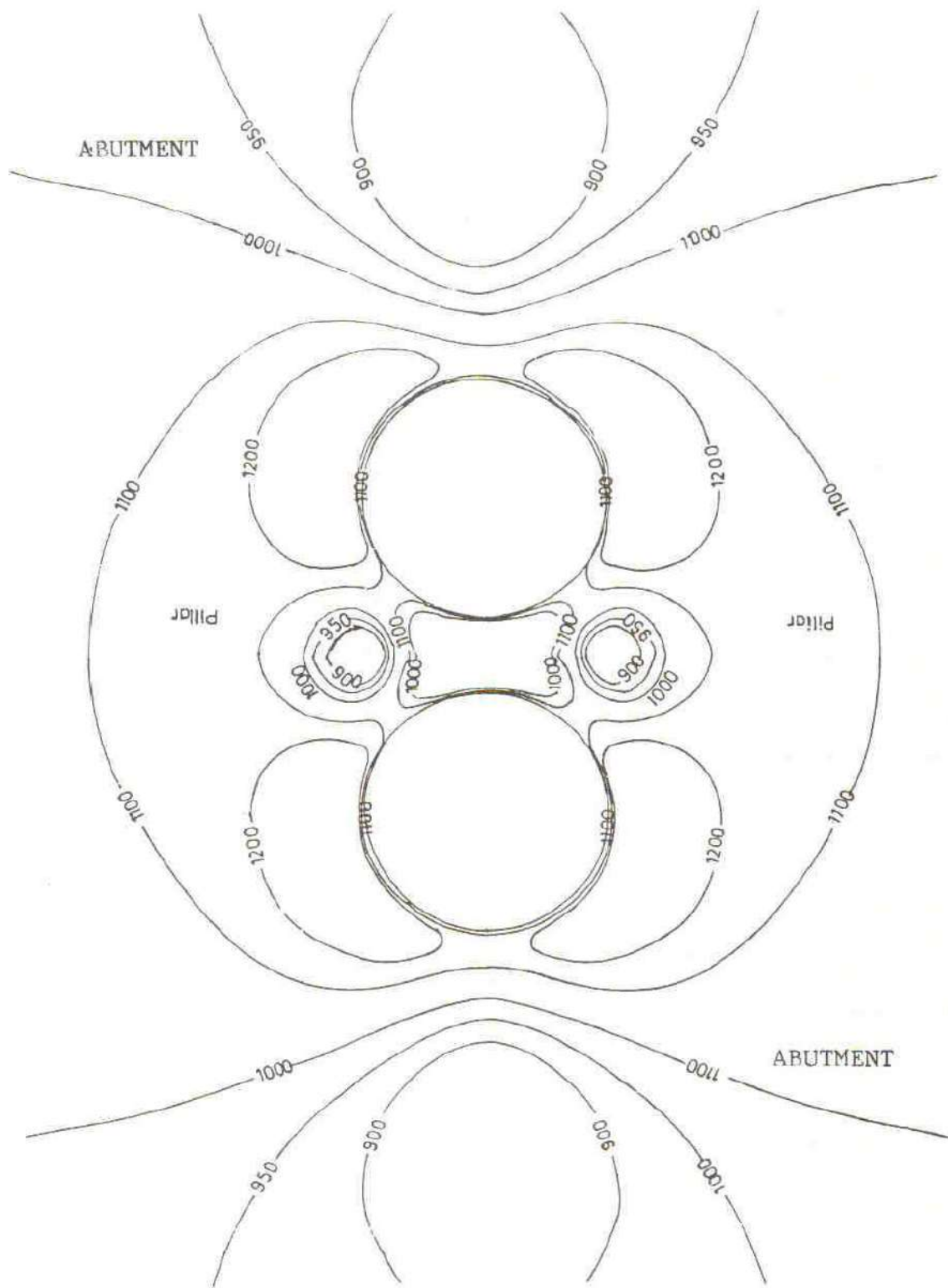


FIGURE 13 Major Principal Stress Contours ( $W/D = 0.3$ ,  $K_0 = 1.5$ ) — EP Analysis

## 5.5 Comparison of Displacements in Interacting Tunnels

### 5.5.1 Comparison of displacements - interacting and single tunnels

The variation of the percentage difference in displacements with W/D for the three in-situ stress ratios have been analysed.

At abutment, the percentage difference is positive for all the three in-situ stress ratios and this means that, the interaction of tunnels shows more horizontal displacement than the single tunnel at this location. The percentage difference of displacements as well as the rate of decrease of percentage difference are high for in-situ stress ratio of 1.0 as compared to other two stress ratios. Lower is the stress ratio, higher is the stress concentration at this location and this has resulted in more yielding in this case. At crown, the decrease of percentage difference is higher for  $K_0 = 0.5$ , but the rate of decrease in percentage difference has been larger here as compared to those at abutment side. At springing level on the pillar side,  $K_0 = 0.5$  shows positive difference, which means interaction causes large horizontal displacement. In the case of in-situ stress ratio of 1.0, at  $W/D = 0.3$ , the percentage difference is positive, but it becomes negative and then increases in the negative direction with increasing W/D ratio. In this case, for this in-situ stress ratio it is evident that, at lower pillar widths, interaction causes more displacement and at larger pillar widths, interaction shows less displacements. Further increase in W/D ratio is likely to reduce the percentage difference. For the in-situ stress ratio of 1.5, the percentage difference is negative, indicating lower yielding due to interaction. In this case also, the percentage difference increases with W/D ratio and has a tendency to decrease with further increase in W/D ratio. At this location, The variation in percentage difference is largest. Among the three in-situ stress ratios,  $K_0 = 0.5$  shows the maximum variation.

The variation of the ratio of change in diameter at springing level and at crown-invert with W/D ratio for elasto plastic analysis have been studied. It is observed from a comparison of ratio of change in diameter in interacting and single tunnels, that when in-situ stress ratio is other than 1, more distance is required between the two tunnels (i.e. higher W/D ratio) to minimise the interaction effects.

### 5.5.2 Comparison of displacements in interacting tunnels - SQEP & SMEP analyses

Figure 14 shows, the variation of percentage difference in displacement between sequential and simultaneous excavation of interacting tunnels with W/D ratio. The percentage difference for both, right hand side tunnel (excavated first) and the left hand side tunnel excavated in the second stage are plotted.

At springing level on the abutment side (Fig. 14a), the percentage difference in displacements is relatively small. Among the two tunnels, the percentage difference is generally higher for

the RHS tunnel. With increasing W/D ratio, the difference show a tendency to become zero. Three distinct stages of interaction can be identified. Because, with increasing W/D ratio, the percentage difference reaches zero and then again, it becomes positive or negative. With further increase in W/D ratio, it is expected to reach zero.

At crown (Fig. 14b), the percentage difference for the in-situ stress ratio,  $K_0 = 0.5$  is almost zero. This is expected also because for this stress ratio, no yielding is indicated near the crown. Maximum difference is indicated for in-situ stress ratio of 1.5. Among the two

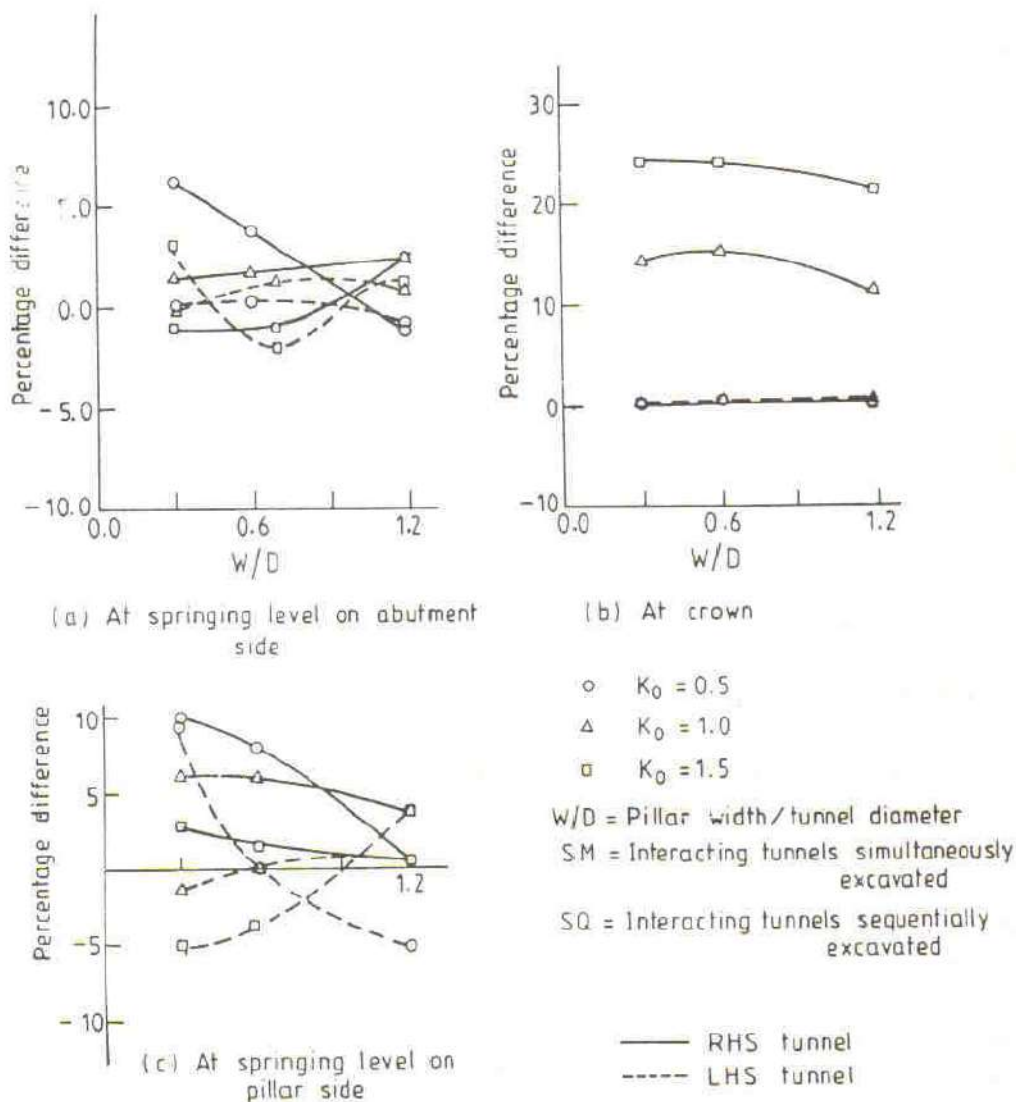


FIGURE 14 Variation of Percentage Difference in Displacements between SQMP and SMEP with W/D



tunnels, maximum difference in percentage difference is indicated for in-situ stress ratio 1.5 and minimum difference for  $K_o = 0.5$ . The percentage difference for  $K_o = 1.0$  is in between the two. With increase in W/D ratio, the percentage difference shows a decreasing tendency and is expected to become zero.

At springing level on the pillar side (Fig. 14c), with increasing W/D ratio, for  $K_o = 0.5$ , the percentage difference decrease from high positive value and is expected to become zero. For  $K_o = 1.5$ , the percentage difference in the RHS tunnels, shows a decrease with increase in W/D ratio. In the case of LHS tunnel, the percentage difference, shows first an increase from negative value, then it becomes positive and again shows a tendency to decrease and is expected to reach zero difference at large W/D ratio. The variation in the case of  $K_o = 1.0$  lies in between that of  $K_o = 0.5$  and 1.5 cases and in this case also the percentage difference shows a tendency to become zero with increase in W/D ratio. In general, the percentage difference are higher for the RHS tunnel as compared to LHS tunnel. In this case also, three distinct stages of interaction can be identified because, with increasing W/D ratio, percentage difference changes its sign once and is expected to become finally zero at large W/D ratio, where the sequential or simultaneous excavation makes no difference.

The variation of the ratio of change in diameter along the springing level and along crown-invert have been computed for three W/D ratio and are found to be almost same having value of approximately unity, for both the tunnels, indicating that for the present set of properties the overall difference in sequential and simultaneous excavations is marginal. The maximum effect of the two types of excavation schemes is observed for higher in-situ stress ratio ( $K_o = 1.5$ ) for closer spacing of tunnels ( $W/D = 0.3$ ).

## 5.6 Comparison of Principal Stresses in Interacting Tunnels

### 5.6.1 Comparison of principal stresses - Interacting and single tunnels

Figure 15 shows the variation of percentage difference in principal stresses with W/D ratio at tunnel boundary, near the tunnel boundary (where maximum major principal stress is observed) and at the centre of pillar zone, for the three in-situ stress ratios.

At the tunnel boundary (Fig. 15a), the percentage difference in the case of major principal stress is negative for  $W/D = 0.3$ , it tends to become zero for in-situ stress ratio of 0.5 and 1.0 with increase in W/D ratio. But it is interesting to note that for  $K_o = 1.5$ , with increasing W/D ratio, the percentage difference becomes zero from a negative value of  $W/D = 0.3$  and then becomes positive with further increase in W/D ratio. It is expected to come down again and reach zero percentage difference as W/D ratio is further increased.

Near the tunnel boundary (Fig. 15b), maximum percentage difference is indicated in the case

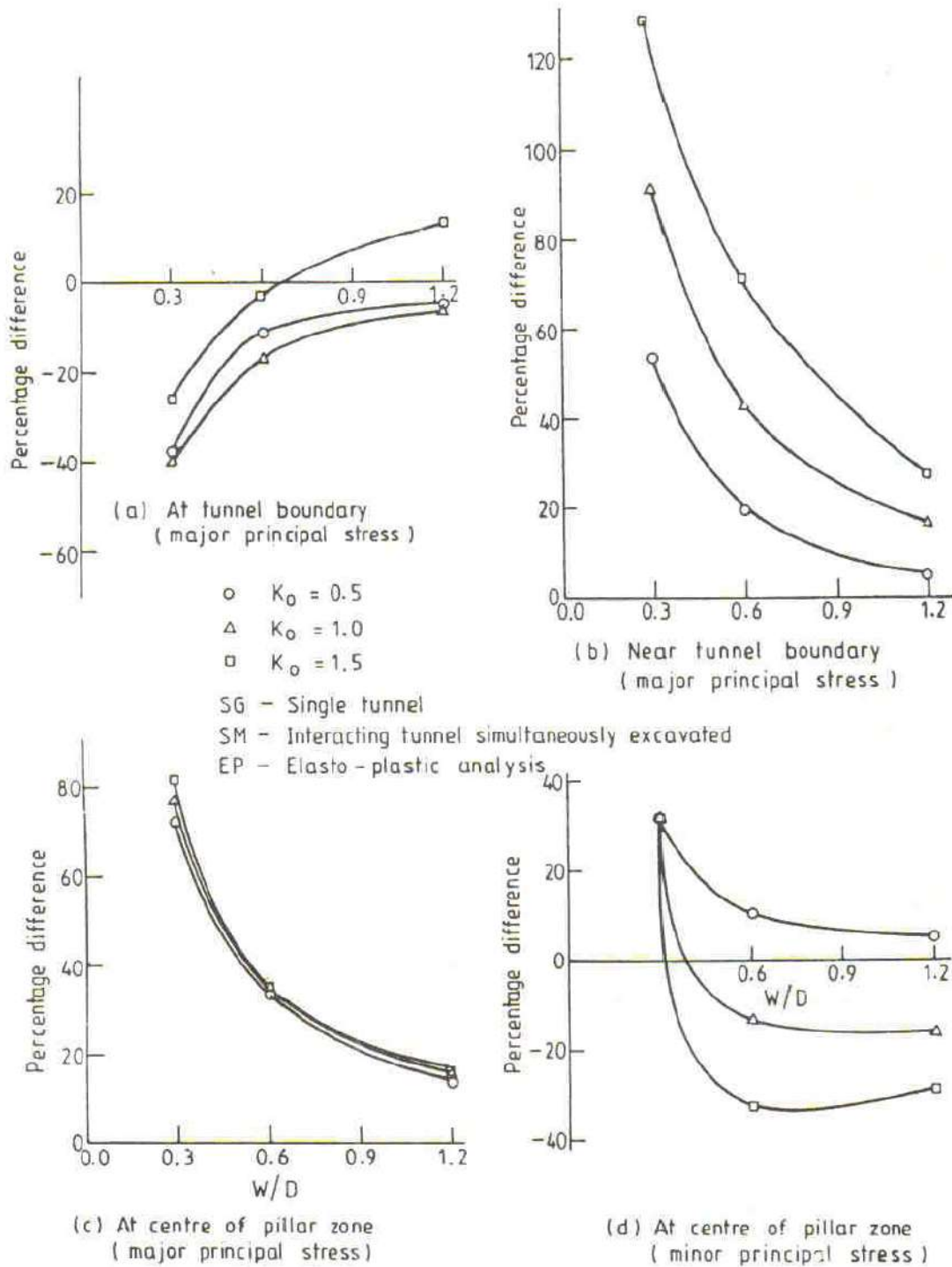


FIGURE 15 Variation of Percentage Difference in Principal Stresses between SMEP and SGEP with W/D



of smaller W/D ratio of 0.3 and this decreases as W/D ratio is increased. Among the three in-situ stress ratios, the percentage difference is higher for  $K_0 = 1.5$ , and lower for  $K_0 = 0.5$ . The rate of decrease in percentage difference with increase in W/D ratio is largest for  $K_0 = 1.5$  and comparatively smaller for  $K_0 = 0.5$ .  $K_0 = 1.0$  case remains in between. With increase in W/D ratio, the percentage difference shows a tendency to reach zero level at large W/D ratio when interaction effects would die down.

At the centre of pillar zone (Fig. 15c), the difference in percentage difference for the three in-situ stress ratios is rather small. In general, the percentage differences are high at smaller W/D ratios and they decrease with increasing W/D ratio and tend towards zero as expected.

The percentage difference in minor principal stress at the centre of the pillar zone is shown in Fig. 15d. In this case, at W/D = 0.3, the percentage difference are positive and for  $K_0 = 0.5$  the difference decrease from higher positive value at smaller W/D ratio towards zero as W/D ratio increases. For  $K_0 = 1.0$  and 1.5, it is interesting to note that at W/D = 0.3, the percentage difference is positive but with increase in W/D ratio, it becomes zero and then negative. With further increase in W/D ratio it is expected to show a tendency to reach zero percentage difference level.

### 5.6.2 Comparison of principal stresses in interacting tunnels - SMEP & SMEL analyses

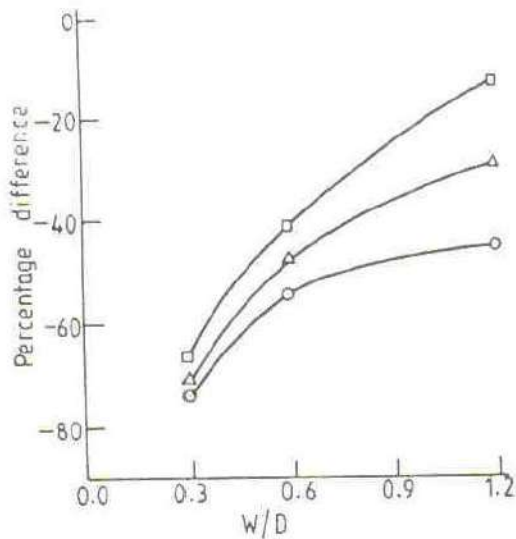
Figure 16 shows the variation of percentage difference in principal stresses between the elasto-plastic and elastic analyses of simultaneously excavated tunnels for the three in-situ stress ratios.

At the tunnel boundary (Fig. 16a) the percentage difference in major principal stress is a larger negative value (indicating that the stresses are higher for elastic analysis) for W/D = 0.3 which decreases with increases in W/D ratio (smaller negative value) and tends to become constant and reach the level of percentage difference indicated for the case of single tunnels.

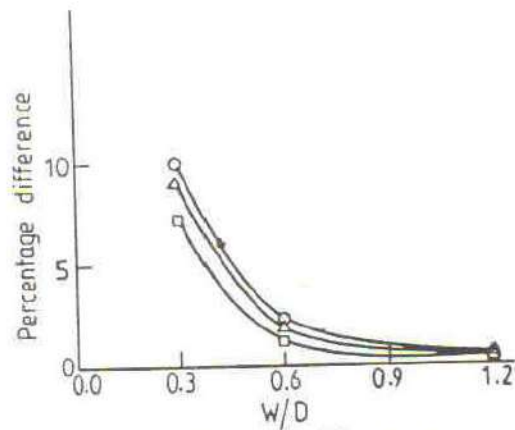
At the centre of pillar zone (Fig. 16b), the difference in the percentage difference is small and it starts decreasing from a comparatively higher value to a constant value as W/D increases.

The variation of percentage difference in the case of minor principal stress at the centre of pillar zone is shown in Fig. 16c. At W/D ratio of 0.3, the percentage difference is comparatively higher and positive and it decreases with increase in W/D ratio at a faster rate. In this case also, the percentage difference tends to become constant at the level of percentage difference for single tunnels.

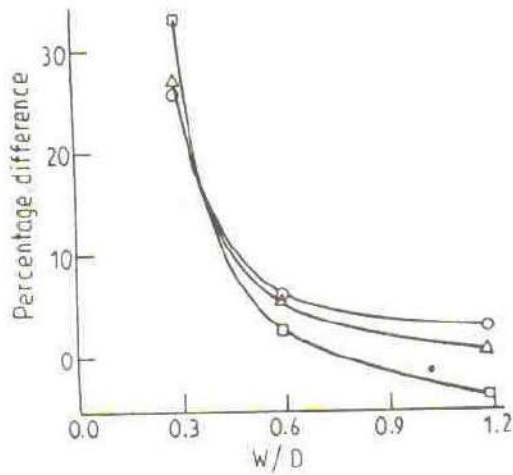




(a) At tunnel boundary.  
(major principal)



(b) At centre of pillar zone  
(major principal stress)



(c) At centre of pillar zone  
(minor principle stress)

- K<sub>0</sub> = 0.5
- △ K<sub>0</sub> = 1.0
- K<sub>0</sub> = 1.5
- SM - Interacting tunnel simultaneously excavated
- EL - Elastic analysis
- EP - Elasto-plastic analysis

FIGURE 16 Variation of Percentage Difference in Displacements between SMEP and SMEL with W/D

## 6. CONCLUSIONS

Analysis of deep single and interacting tunnels have been carried out in the present work. Some typical results have been presented. Overall conclusions from various analyses are summarised as follows :

- (i) In the case of single tunnels for elastic analysis, the displacement are very much a function of in-situ stress ratio and the maximum displacement noted at the boundary is more for stress ratio other than 1 (near the springing if  $K_0 > 1$  and near crown if  $K_0 < 1$ ). In the case of elasto-plastic analysis, the displacement distribution at the tunnel boundary tends to be comparatively uniform for in-situ state of stress condition other than hydrostatic.
- (ii) If the material behaviour is elasto-plastic, the sequence of excavation has significant effect on the final stress and displacements around the tunnel. Also, these are comparatively non-uniform in the case of two stage excavation as compared to single stage excavation. The two stage excavation scheme analysed in the present case for circular and horse-shoe tunnels, results in development of tensile stresses near the springing level of the opening and it causes reveals of stresses and direction of movement of points around the tunnel.
- (iii) Among the two shapes of single tunnels analysed, it has been found that for  $K_0 > 1$ , less yielding and displacements are obtained for circular tunnels as compared to horse-shoe tunnels and for  $K_0 < 1$ , reverse trend is observed. The horse-shoe shaped tunnel has been found to be less preferable for higher in-situ stress ratio.
- (iv) In the case of interacting tunnels, it has been found that the interaction effect increase with decreasing  $W/D$  ratio for all in-situ stress conditions. This interaction effect is higher for smaller in-situ stress ratio ( $K_0 = 0.5$ ), if the material behaviour is elastic. In case th material behaviour is elasto-plastic, the interaction effects are greater for the higher in-situ stress ratio ( $K_0 = 1.5$ ). In general, the interaction effect is very much a function of in-situ stress condition along with the space between the tunnels. The spacing required to reduce the interaction effect is higher for higher in-situ stress ratios ( $K_0 = 1.5$ ). With increase in  $W/D$  ratio, the change from interacting to non-interacting (single) opening behaviour is not smooth.

## 7. REFERENCES

- Chandrasekaran, V.S. and King G.J.W. (1974): "Simulation of Excavation Using Finite Elements", JI of Geotech. Eng. Div., ASCE, Vol. 100, pp. 1087-1089.

- Hoek, E. and Brown, E.T. (1980) : "Underground Excavation in Rock", Institution of Mining and Metallurgy, London.
- Nayak, G.C. and Zienkiewicz, P.C. (1972) : "Convenient Form of Stress Invariants for Plasticity", *Jl Structural Eng. Div. ASCE*, Vol. 98, pp. 949-954.
- Zienkiewicz, O.C. and Corneau, J.C. (1974) : "Viscoplasticity, Plasticity and Creep in Elastic Solids : A Unified Numerical Solution Approach", *Int. Jl. Num. Meth. Engg.* Vol. 8, pp. 821-845.