A Statistical Approach to Landslide Analysis

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ABSTRACT

The approach of Hoek & Bray (1981) to analyse the plane failure analysis is modified to take into account the release joint inclination. Accordingly, the equations to calculate the factor of safety and other parameters have been changed. This modified approach has been subjected to statistical tools and simple equations have been proposed for direct estimation of the factor of safety of plane failure of slopes. These new equations are obtained using the multiple regression analysis. Adequacy of these equations has been tested by applying F Test, Durbin-Watson Test and Heteroscedasticity and Homoscedasticity Tests.

1. INTRODUCTION

The limit equilibrium approach of rock slope stability analysis is a well established procedure. This method is essentially based on the sliding friction block model and assumes that the failed rock mass slides along a failure surface if the disturbing forces exceed the resisting forces. This failure surface can be wedge shaped or planar, depending on the orientation of joints or joint sets, in rock mass. However, the limit equilibrium method, is generally an elaborate and cumbersome technique to use, in spite of various readily accessible aids for calculation. Hence, a single formula for estimating the factor of safety, if evolved based on statistical approach may have greater field applications.

This paper proposes new equations for the estimation of factor of safety for critical plane failures of dry and wet slopes, by using the multiple regression analysis. However, before going into the detailed statistical analysis, it is necessary to discuss the technique of plane failure analysis.

2. MODIFIED TECHNIQUE OF PLANE FAILURE ANALYSIS

The approach of Hoek & Bray (1981) is commonly preferred to analyse the plane failure mode in rock slopes. In this approach it has been assumed that the upper slope surface is horizontal and the tension crack is vertical. Though this approach is useful in analysing slopes of open cast mines but it is not of much use for hilly slopes. During the field work related to this research programme, it has been observed that the angle of upper slope is either equal or slightly less and seldom greater than the angle of failed slope. Moreover the position and depth of tension crack is very difficult to trace out on the upper slope surface or on slope face. If a vertical or near vertical set of discontinuity is not present then one has to assume a arbitrary tension crack or do the analysis without considering the effect of tension crack. Thus, keeping these observations in mind, an attempt has been made to analyse the plane failure mode by assuming that any major or minor discontinuity, whatever its inclination be and dipping into the slope, act as the release joint. Subsequently, the equations to calculate the factor of safety and other parameters are changed.

The geometry of the slope considered in the present analysis is defined in Fig. 1. The various symbols used in the figure are

- α_f, Slope surface angle - α_p, Dip of potential failure plane - α_t, Angle of release joint - L, Inclined distance of release joint from toe of slope - h, Height of release joint from toe of slope	- H, Height of slope - Z, Depth of release joint - W, Weight of the sliding block - U, Uplift water force acting on the block - V, Water force in the tension crack, acting on the rear face of the block

The equations to calculate the Area, Weight, Water Pressure and Factor of safety is given below:

Area of the Sliding Block

$$A = L^*[\mathrm{Sin}(\alpha_f+\alpha_t)] \;/\; \mathrm{Sin}(\alpha_p+\alpha_t)$$

Weight of the Sliding Block

$$(W) = \tfrac{1}{2}\,\gamma\, L^2\, [\mathrm{Sin}(\alpha_f+\alpha_t)\ \mathrm{Sin}(\alpha_f-\alpha_p)] \,/\, \mathrm{Sin}(\alpha_p+\alpha_t)$$

Vertical Depth of Release Joint

$$Z = L\ \mathrm{Sin}\alpha_t\ [\mathrm{Sin}(\alpha_f-\alpha_p)] \,/\, \mathrm{Sin}(\alpha_p + \alpha_t)$$

Horizontal Water Force $(V) = \tfrac{1}{2}\,\gamma_w Z_w^2$

Uplift Water Force $(U) = U = \tfrac{1}{2}\,\gamma_w Z_w A$

where, γ_w = Unit weight of water

Z_w = Depth of water in release joint

If the dip of release joint varies between 10° to 60°, the factor of safety is calculated by the following equation

$$F = CA + [\ (W\ \mathrm{Cos}\ \alpha_p - U)\ \mathrm{Tan}\ \phi]/W\ \mathrm{Sin}\ \alpha_p + V$$

On the other hand, the factor of safety for slopes in which the dip of release joint varies from 61° to 90° is obtained from equation

$$F = CA + [(W\ \mathrm{Cos}\ \alpha_p - U - V\ \mathrm{Sin}\ \alpha_p)\ \mathrm{Tan}\ \phi]/W\ \mathrm{Sin}\ \alpha_p + V\ \mathrm{Cos}\ \alpha_p$$

The inclined distance, L, of release joint measured from toe of slope is not easy to measure in the field, as the release joint would be well distributed over the slope face. Now, without knowing this distance, factor of safety cannot be calculated. Therefore the value of L can be varied from minimum to maximum possible distance during analysis. This type of analysis would ensure the particular height of slope from where the slope is unstable.

Thus, nine slopes have been analysed for plane mode of failure by using the above discussed modified technique of plane failure. Of these nine slopes, eight slopes fall within the reservoir area of Lakhwar dam and one falls on the right abutment slope at Lakhwar dam site. The input data and the corresponding factor of safety under no earthquake loading for each plane failure analysis is shown in Table1.

3. STATISTICAL ANALYSIS

3.1 Correlation analysis

With a view to investigate the relationship, if any, between the factor of safety of plane failure and parameters (used to determine Factor of safety), Pearson's correlation coefficient has been computed, using the formula given below

$$r = \Sigma[(X_i - X_m)(Y_i - Y_m)] \, / \, [\Sigma(X_i - X_m)^{*}\Sigma(Y_i - Y_m)]^2$$

Where

r = correlation coefficient between the two parameters X and Y,
X_i = ith observation of parameter X,
Y_i = ith observation of parameter Y,
X_m = mean of parameter X,
Y_m = mean of parameter Y, and
n = number of observations

TABLE 1 INPUT DATA SHEET FOR CORRELATION ANALYSIS

Section No.	Dependent Variable				Independent Variable					
	Factor of Safety				α_f	α_p	α_t	C	ϕ	h
	F_1	F_2	F_3	F_4						
YR1(A)	0.97	0.92	0.88	0.78	60	51	40	9.69	30	104
YL4(C)	1.10	0.99	0.96	0.81	61	50	54	15.62	30	119
YL6(A)	1.46	1.41	1.34	1.22	74	71	41	8.61	40	104
YR11(A)	0.90	0.85	0.81	0.63	60	53	60	7.53	30	104
YL14(B)	1.27	1.25	1.21	1.16	65	62	27	18.87	30	296
YR17(A)	1.04	1.00	0.95	0.85	55	51	41	5.14	30	104
YR4(B)	1.14	1.09	1.03	0.91	45	41	41	4.88	30	119
YL8(A)	0.99	0.87	0.74	0.44	72	62	71	8.44	30	70
Right Abutment	1.19	1.11	1.05	0.89	58	53	46	10.00	40	160

α_f = Dip of Slope Face (deg.)
α_t = Release Joint Inclination (deg.)
α_p = Failure Plane Inclination (deg.)
F_1 = Dry Release Joint
F_2 = Release Joint Filled Quarter With Water
h = Height of Release Joint (m)
C = Cohesion ($t\ m^{-2}$)
ϕ = Angle of Internal Friction (deg.)
F_3 = Release Joint Filled Half with Water
F_4 = Release Joint Filled Full with Water

The factor of safety of plane failure, is dependent on the following parameters

α_f, Slope face inclination
α_p, Failure plane inclination
α_t, Release joint inclination
c, cohesion
ϕ, Angle of Internal Friction
h, Height of release joint

It is clearly evident from the method that the correlation of factor of safety with difference of slope face inclination and failure plane inclination (α_f - α_p) is more important rather than its correlation with slope face inclination (α_f) and Failure plane inclination (α_p) separately. Therefore, instead of obtaining the correlation of Factor of safety with α_f and α_p separately, the correlation of (α_f - α_p) with Factor of safety has been obtained.

It is clear from the Table 1 that factor of safety is highly sensitive to the depth of water in release joint. Thus, correlation analysis has been carried out in all four conditions.

i)	Release joint dry = F_1	ii)	Quarter-filled with water = F_2
iii)	Half-filled with water = F_3	iv)	Fully-filled with water = F_4

The correlation coefficient were calculated for all possible 15 pairs of variables from 9 slopes and are presented in Table 2. These coefficients were also tested for their statistical significance. These coefficients are also shown in the form of bars (Fig.2).

3.1.1 *Interpretation of Correlation Coefficient* :

A perusal of Table 2 indicates that factor of safety (F_1, F_2, F_3, F_4) shows high negative correlation of - 0.63, - 0.73, -0.73 and - 0.75 with α_f - α_p at significant level of 5%, 2.5%, 2.5% and 2.5% respectively. Such a correlation is natural to obtain as more is the difference of slope face inclination and failure plane inclination less is the factor of safety. It also shows negative correlation with release joint inclination suggesting the close association of release joint inclination and Factor of safety. The factor of safety also show positive correlation with angle of internal friction and height of release joint from toe of slope.

However, in case of angle of internal friction the significance level is decreasing towards F_4 condition suggesting that a non-linear relation may exists between factor of safety and angle of internal friction in the presence of water. Whereas, in case of height of release joint the significance level is increasing towards F_4 condition with significance level below 10% in F_1 condition which means that height of release joint from toe of slope has more linear relationship with factor of safety in F_4 condition than F_1. However, the correlation coefficient between factor of safety and cohesion is insignificant in all F1,F2,F3 and F4 conditions.

TABLE 2 CORRELATION MATRIX OF FACTOR OF SAFETY WITH INDEPENDENT VARIABLES

Independent Variable	Factor of Safety			
	F_1	F_2	F_3	F_4
$\alpha_f - \alpha_p$	-0.63 (5%)	-0.73 (2.5%)	-0.73 (2.5%)	-0.75 (2.5%)
α_r	-0.55 (10%)	-0.66 (2.5%)	-0.73 (2.5%)	-0.85 (0.5%)
C	0.29 (<10%)	0.26 (<10%)	0.31 (<10%)	0.33 (<10%)
ϕ	0.68 (2.5%)	0.63 (2.5%)	0.60 (5%)	0.47 (10%)
h	0.41 (<10%)	0.47 (<10%)	0.53 (10%)	0.60 (5%)

Figures in parenthesis represents the level of significance

3.2 Multiple Regression Analysis

In Table 2 it can be seen that the factor of safety is more or less linearly related to the independent variables. The linearity increases towards F_4 condition i.e when release joint is full filled with water. Hence, it was decided to carry out the multiple linear repression to fit a single equation of the form $Y = f(X_1, X_2, ... X_n)$ for plane mode of failure. The model has been predicted for F_1, F_2, F_3, F_4 conditions i.e release joint dry, quarter filled with water, half filled with water and full filled with water.

The development of a new equation for direct estimation of the factor of safety of a critical plane failure has been achieved as follows. Firstly, the factor of safety must follow some logical relation with the geometry of slope and geotechnical properties. This relation can be written as

$$F = f[1/(\alpha_f - \alpha_p), 1/\alpha_r, C, \phi, 1/h]$$

where	
α_f = Slope face inclination (deg.),	C = Cohesion ($t\ m^{-2}$),
α_p = Failure Plane Inclination (deg.),	ϕ = Angle of Internal Friction (deg.),
α_r = Release Joint Inclination (deg.),	h = Height of Release Joint (m)

It is clearly evident from the above equation that i) least the difference of slope face inclination and failure plane inclination more is the factor of safety , ii) steep is the release joint inclination less is the factor of safety iii) more is the value of cohesion more is the value of factor of safety iv) more is the value of angle of internal friction more is the value of factor of safety v) more is the height of slope the value of factor of safety decreases. All relations of independent variables with Factor of safety described above very well matches with the modified technique of plane failure analysis. The data required to obtain the model equations is given in Table 3.

A regression of any m independent variables upon a dependent variable can be expressed as

$$Y = b_0 + b_{1.23456}{}^*X_1 + b_{2.13456}{}^*X_2 + b_{3.12456}{}^*X_3 + b_{4.12356}{}^*X_4 + b_{5.12346}{}^*X_5$$

TABLE 3 INPUT DATA SHEET FOR MULTIPLE REGRESSION ANALYSIS

Section No.	Dependent Variable				Independent Variable				
	Factor of Safety				$1/\alpha_f-\alpha_p$	α_f	C	ϕ	1/h
	F_1	F_2	F_3	F_4					
YR1(A)	0.97	0.92	0.88	0.78	0.1111	0.0250	9.69	30	0.0096
YL4(C)	1.10	0.99	0.96	0.81	0.0909	0.0185	15.62	30	0.0084
YL6(A)	1.46	1.41	1.34	1.22	0.3333	0.0243	8.61	40	0.0096
YR11(A)	0.90	0.85	0.81	0.63	0.1428	0.0166	7.53	30	0.0096
YL14(B)	1.27	1.25	1.21	1.16	0.3333	0.0370	18.87	30	0.0033
YR17(A)	1.04	1.00	0.95	0.85	0.2500	0.0243	5.14	30	0.0096
YR4(B)	1.14	1.09	1.03	0.91	0.2500	0.0243	4.88	30	0.0084
YL8(A)	0.99	0.87	0.74	0.44	0.1000	0.0140	8.44	30	0.0142
Right Abutment	1.19	1.11	1.05	0.89	0.2000	0.0217	10.00	40	0.0062

The b's in the above regression equation are called partial regression coefficients which gives the rate of change in the dependent variable for a unit change in that particular independent variable, provided all other independent variables are held constant. The coefficient $b_{1.2345}$, for example, is read "the regression coefficient of variable 1 on Y, as variable 2,3,4,and 5 remain constant" (Davis,1973).

Thus, the equation for plane failure has been worked out under full filled condition, half filled condition, quarter filled condition and dry condition of release joint. The partial regression coefficient for each independent variable, goodness of fit, multiple correlation coefficient among with analysis of variance (ANOVA) is shown in Table 4.

TABLE 4 STATISTICS OF REGRESSION MODELS

REGRESSION MODEL 1

Variable	Regression Coefficient
$X_1 = 1/\alpha_r - \alpha_p$	1.34744
$X_3 = C$	0.01741
$X_4 = \phi$	0.01819
$X_5 = 1/h$	19.0086
Constant	-0.07893

Standard Error of Estimation = 0.07941

Goodness of Fit $R^2 = 0.89$

Coefficient of Multiple Correlation = 0.95

$$F_1 = -0.0789 + 1.347X_1 + 0.017X_3 + 0.018X_4 + 19.009X_5$$

Analysis of Variance

Source	Sum of Squares	Degree of Freedom	Mean Square	F Test	Probability
Regression	0.21137	4	0.05284		
Residual	0.02523	4	0.00631	8.379	0.0317
Total	0.23660	8			

REGRESSION MODEL 2

Variable	Regression Coefficient
$X_1 = 1/\alpha_r - \alpha_p$	1.38067
$X_2 = 1/\alpha_r$	3.35359
$X_3 = C$	0.01336
$X_4 = \phi$	0.01738
$X_5 = 1/h$	14.84397
Constant	-0.012190

Standard Error of Estimation = 0.08041

Goodness of Fit $R^2 = 0.92$

Coefficient of Multiple Correlation = 0.96

$$F_2 = -0.1219 + 1.381X_1 + 3.354X_2 + 0.013X_3 + 0.017X_4 + 14.884X_5$$

Analysis of Variance

Source	Sum of Squares	Degree of Freedom	Mean Square	F Test	Probability
Regression	0.25062	5	0.05012		
Residual	0.01940	3	0.00647	7.752	0.0612
Total	0.27002	8			

For dry release joint (regression model 1) the variable $1/\alpha_t$ has least importance, as would be later discussed, and therefore has not been taken into account. The standard error of estimation for this model is 0.0794 while the goodness of fit (R^2) or the proportion of total variation about the mean of Y explained by the regression is 0.89. Thus the regression model obtained explains 89% of the total variation. The coefficient of multiple correlation R is 0.95.The standard error of estimation for regression model 2 is 0.08041, where as the goodness of fit and multiple correlation coefficient is 0.93 and 0.96 respectively.

For regression model 3, the standard error of estimation is 0.0898 where as the goodness of fit is 0.91, thus explaining 91% of the total variation, the coefficient of multiple correlation is 0.96. The standard error of estimation, goodness of fit and multiple correlation coefficient for regression model 4 is 0.085, 0.94 and 0.97 respectively. The plots of observed and predicted factor of safety for all conditions are shown in Fig. 3.

REGRESSION MODEL 3		
Variable	Regression Coefficient	Standard Error of Estimation = 0.07941
$X_1 = 1/\alpha_f - \alpha_p$	1.1813	Goodness of Fit R^2 = 0.89
$X_2 = 1/\alpha_t$	3.4810	Coefficient of Multiple Correlation = 0.95
$X_3 = C$	0.0105	
$X_4 = \phi$	0.0179	
$X_5 = 1/h$	1.1565	$F_3 = -0.008 + 1.181X_1 + 3.481X_2 + 0.011X_3 + 0.018X_4 + 1.157X_5$
Constant	-0.0080	

Analysis of Variance

Source	Sum of Squares	Degree of Freedom	Mean Square	F Test	Probability
Regression	0.2650	5	0.0530		
Residual	0.0242	3	0.0081	6.574	0.0760
Total	0.2892	8			

The significance of the regression model is also been tested by performing F-test through the analysis of variance (ANOVA). In more formal terms the F-test for significance of fit is a test of the null hypothesis and alternative

$H0 : ß1 = ß2 = ß3 = ß4 = ß5 = 0$
$H1 : ß1, ß2, ß3, ß4, ß5 = 0$

The hypothesis to be tested is that the partial regression coefficient are equal to zero, or in other words, there is no regression. If the computed value of F exceeds the selected table value of F, this hypothesis is rejected and the alternative H1 is accepted (Draper & Smith, 1966 and Davis, 1973).

For regression model 1, the calculated value of F is 8.349. Now we will use a 5% level of significance (α = 0.05). The test statistics follows an F distribution with degrees of freedom $\mu 1 = 4$ and $\mu 2 = 4$, so the critical region would consists of values exceeding

REGRESSION MODEL 4					
Variable	Regression Coefficient	Standard Error of Estimation = 0.07941			
$X_1 = 1/\alpha_f - \alpha_p$	1.02332	Goodness of Fit R^2 = 0.89			
$X_2 = 1/\alpha_t$	11.85309	Coefficient of Multiple Correlation = 0.95			
$X_3 = C$	0.00956				
$X_4 = \phi$	0.02007				
$X_5 = 1/h$	3.66832	$F_4 = -0.3708 + 1.023X_1 + 11.853X_3 + 0.010X_3 + 0.020X_4 + 3.668X_5$			
Constant	-0.37084				
Analysis of Variance					
Source	Sum of Squares	Degree of Freedom	Mean Square	F Test	Probability
Regression	0.34959	5	0.06992		
Residual	0.02170	3	0.00723	9.665	0.0455
Total	0.37129	8			

F = 6.39. The computed test value (F = 8.39) fall in the critical region and we can reject the hypothesis $H0 : ß1 = ß2 = ß3 = ß4 = ß5 = 0$ running a risk of less than 5% of being wrong.

Similarly, for regression model 4 the calculated value of F exceeds the tabulated value of F at 5% level of significance i.e. $F = 9.665 > F_{5,3,.95}$ = 9.01. Thus, the hypothesis of no regression can be rejected.

However, the regression model 2 & 3 instead of 5% level of significance can only be rejected on 10% level of significance.

Thus, it can be concluded that for regression model 1 and 4, the hypothesis of no regression can be rejected when it is correct 1 times out of 20 trials while for regression model 2 & 3 the hypothesis can be rejected when it is correct 1 times out of 10 trials.

3.2.1 *Adequacy of regression models*

Durbin-Watson test has been applied to see whether the deviations or residuals from the regression are free from autocorrelation, an assumption made in regression analysis. Autocorrelation in this context means that residuals tend to occur as "clumps" of adjacent deviations on the same side of the regression line. Durbin-Watson statistic, which always lie between 0 and 4, states that the values close to 0 indicate positive autocorrelation, values close to 4 indicate negative autocorrelation, and values close to 2 indicate that residuals are free from autocorrelation (Chatterjee & Price, 1977).

Another way of checking the regression equation is that whether the standardized residuals are following the normal distribution (mean = 0, standard deviation = 1) or not. Further, about 99.7% of the standard residuals should fall between + 3 and - 3; 95% of the standardized residuals should fall between +2 and -2; and 68% of the standardized residuals should fall between +1 and -1 (Chatterjee & Price, 1977) . The Durbin-Watson statistic, residuals, standardized residuals and their plots, for F_1, F_2, F_3, and F_4 release joint conditions are shown in Table 5.

TABLE 5 EXAMINATION OF REGRESSION MODELS

REGRESSION MODEL 1						
Factor of Safety			Standardized Residual			
Observed	Predicted	Residual	Value	Mean	Std. Dev.	Durbin Watson Test
0.97	0.968	0.00244	0.043452	$-1.98*10^{-5}$	1.00	2.01
1.10	1.021	0.07921	1.410590			
1.46	1.430	0.02998	0.533890			
0.90	0.973	-0.07267	-1.29412			
1.27	1.307	-0.03705	-0.65979			
1.04	1.075	-0.03550	-0.63219			
1.14	1.048	0.09184	1.635508			
0.99	1.018	-0.02828	-0.50361			
1.18	1.210	-0.02998	-0.53389			

The table shows that for the regression model 1 and (F_1) and regression model 2 (F_2) Durbin-Watson statistic is 2 and 1.87 respectively while for regression model 3 (F_3) and regression model 4 (F_4) the Durbin-Watson statistic is 1.46 and 1.57 respectively. These values suggest that the regression model 1 and 2 is entirely free from autocorrelation but there may exist a little bit of autocorrelation in regression model 3 and 4.

REGRESSION MODEL 2						
Factor of Safety		Residual	Standardized Residual			
Observed	Predicted		Value	Mean	Std. Dev.	Durbin Watson Test
0.92	0.909	0.01143	0.232112	$-1.98*10^{-5}$	1.00	1.87
0.99	0.921	0.06969	1.415217			
1.41	1.372	0.03767	0.764977			
0.85	0.895	-0.04531	-0.92012			
1.25	1.285	-0.03476	-0.70588			
1.00	1.037	-0.03720	-0.75543			
1.09	1.016	0.07409	1.504570			
0.87	0.908	-0.03794	-0.77046			
1.11	1.148	-0.03767	-0.76497			

REGRESSION MODEL 3						
Factor of Safety		Residual	Standardized Residual			
Observed	Predicted		Value	Mean	Std. Dev.	Durbin Watson Test
0.88	0.860	0.0197	0.358313	$2.00*10^{-4}$	1.00	1.46
0.96	0.875	0.0850	1.546022			
1.34	1.288	0.0521	0.947620			
0.81	0.846	-0.0357	-0.64932			
1.21	1.254	-0.0441	-0.80211			
0.95	0.974	-0.0240	-0.43652			
1.03	0.970	0.0602	1.094947			
0.74	0.801	-0.0610	-1.10949			
1.08	1.132	-0.0521	-0.94762			

The standardized residuals of all four regression models are following the normal distribution (mean = 0, standard deviation = 1).Further, about 78% and 100% of standardized residuals are falling in between +1 and -1 in regression model 1 and 2 while in regression model 3 and 4 100% and 89% of standardized residuals are falling in between +1 and -1.

REGRESSION MODEL 4						
Factor of Safety			Standardized Residual			
Obse-rved	Predi-cted	Residual	Value	Mean	Std. Dev.	Durbin Watson Test
0.78	0.769	0.01108	0.212740			
0.81	0.723	0.08650	1.660833			
1.22	1.178	0.04186	0.803727			
0.63	0.681	-0.05112	-0.98152			
1.16	1.203	-0.04294	-0.82446	$2.20*10^{-8}$	1.00	1.58
0.85	0.859	-0.00914	-0.17549			
0.91	0.852	0.05774	1.180630			
0.58	0.632	-0.05212	-1.00072			
0.97	1.012	-0.04186	-0.80372			

Another assumption of linear regression is that the variance is constant about the regression line. This assumption has also been tested by examining the residuals from the fitted line. If the variance is constant, the residuals will form a more or less uniform band around the regression line, a condition known as homoscedasticity (Davis, 1973). If there is a progressive change in the width of the band of deviation, the variance may not be constant, a condition known as heteroscedasticity (Davis, 1973). Thus, the residuals shown in Table 5 has been plotted against calculated factor of safety (Fig.3).

A careful glance of figure shows that for quarter filled (b), half filled (c) and full filled (d) conditions, the residuals are forming horizontal band which indicates no abnormality, and the equations does not appear to be invalidated. However, for dry condition (a) the band of residual plot is not horizontal but is slightly converging indicating that the variance is not constant.

3.2.2 *Relative importance of different X - variables*

The relative effectiveness of the independent variables as predictors of the dependent variable cannot be determined from a direct examination of the regression coefficients. However, by standardizing the partial regression coefficients by converting them to units of standard deviation, they may be compared directly with each other and can be ranked (ignoring sign) in order of the sizes of the standard partial regression coefficient (Davis, 1973).

The standard partial regression coefficient of all independent variables have been obtained in all four conditions (i.e. release joint is dry, quarter filled, half filled and full filled with water) for which the procedure described by Davis, 1973 has been adopted. The equations to calculate the standard partial regression coefficient are

$$[r_{XX}]^*[B_K] = [r_{XY}]$$
$$\text{or } [B_K] = [r_{XX}]^{-1*}[r_{XY}]$$

B = Standard partial regression coefficient

$[r_{XY}]$ = Column vector of correlations between Y and the X_K independent variables

$[r_{XX}]$ = m x m matrix of correlations between the X_K independent variables

In our case, normal equation for five independent variables has the form

$$\begin{matrix} 1 & r_{X1X2} & r_{X1X3} & r_{X1X4} & r_{X1X5} \\ r_{X2X1} & 1 & r_{X2X3} & r_{X2X4} & r_{X2X5} \\ r_{X3X1} & r_{X3X2} & 1 & r_{X3X4} & r_{X3X5} \\ r_{X4X1} & r_{X4X2} & r_{X4X3} & 1 & r_{X4X5} \\ r_{X5X1} & r_{X5X2} & r_{X5X3} & r_{X4X5} & 1 \end{matrix} \quad \begin{matrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{matrix} \quad = \quad \begin{matrix} r_{X1Y} \\ r_{X2Y} \\ r_{X3Y} \\ r_{X4Y} \\ r_{X5Y} \end{matrix}$$

The m x m matrix of correlation in between the independent variables and the column vector of correlations between Factor of Safety in all four conditions and independent variables is given in Table 6.

After obtaining the standard partial regression coefficient the independent variables have been ranked in descending orders (of the sizes of SPRC) for all four conditions (Table 7).

A perusal of Table 7 indicates that α_i - α_p is the most important variable in all regression models and therefore ranked first. In regression model 1 release joint inclination is least important which is in accordance with the factor of safety equation for dry condition in which release joint inclination is not at all used in the calculations. However , its order of

TABLE 6 MATRIX OF CORRELATION BETWEEN FACTOR OF SAFETY AND X_K VARIABLE AND IN BETWEEN X_K INDEPENDENT VARIABLE

	F_1	F_2	F_3	F_4
$1/\alpha_f-\alpha_p$	0.78	0.86	0.86	0.86
$1/\alpha_t$	0.52	0.63	0.68	0.79
C	0.29	0.26	0.31	0.33
ϕ	0.68	0.63	0.60	0.47
1/h	-0.44	-0.51	-0.60	-0.70

	$1/\alpha_f-\alpha_p$	$1/\alpha_t$	C	ϕ	1/h
$1/\alpha_f-\alpha_p$	1.00				
$1/\alpha_t$	0.74	1.00			
C	0.06	0.46	1.00		
ϕ	0.39	0.01	-0.07	1.00	
1/h	-0.54	-0.79	-0.60	-0.60	1.00

importance increases to 4th and 2nd place in regression model 3 and regression model 4 respectively. Angle of internal portion and cohesion remains on 2nd and 3rd place respectively for regression models 1, 2 and 3 but decreases in order of importance to 3rd and 4th place for regression model 4. The height of release joint from toe of slope is the least important factor in regression model 3rd and 4th while it is on 4th place in regression model 1 and 2.

TABLE 7 ORDER OF IMPORTANCE OF INDEPENDENT VARIABLES

RANK	Regression Model 1		Regression Model 2		Regression Model 3		Regression Model 4	
	S	V	S	V	S	V	S	V
1	0.74	$\alpha_f-\alpha_p$	0.75	$\alpha_f-\alpha_p$	0.64	$\alpha_f-\alpha_p$	0.44	$\alpha_f-\alpha_p$
2	0.48	ϕ	0.41	ϕ	0.38	ϕ	0.35	α_t
3	0.46	C	0.34	C	0.28	C	0.30	ϕ
4	0.31	h	0.26	h	0.12	α_t	0.20	C
5	$0.52^* 10^{-4}$	α_t	0.10	α_t	0.06	h	-0.07	h

S = Standard Partial Regression Coefficient
V = Independent Variable

4. CONCLUSION

The existing limit equilibrium method for slope analysis of plane failure is elaborate and time consuming. This problem can be overcome by using the proposed equations for direct estimations of factor of safety for plane mode of failure of rock slopes. These equations have been derived from both failed and stable cases of slopes using the multiple egression analysis. As seen from the equations (Table 4 & 5), there is good agreement between the factor of safety obtained by these equations and by the limit equilibrium method. These equtions has been tested, through ANOVA, that for regression model 1 and 4, the hypothesis of no regression can be rejected when it is correct 1 times out of 20 trials while for regression model 2 & 3 the hypothesis of no regression can be rejected when it is correct 1 times out of 10 trails.

The Durbin-Watson statistics suggests that the regression models 1 and 2 are free from autocorrelation but there may exist a little bit of autocorrelation in regression models 3 and 4. It has been shown that for F_2, F_3 and F_4 conditions, the residuals are forming horizontal band which indicates no abnormality, and the equations does not appear to be invalidated. However, for dry condition (F_1) the band of residual plot is not horizontal but is slightly converging indicating that the variance is not constant.

The difference of slope face inclination and failure plane inclination is the most important variable in all regression models. In regression model 1 release joint inclination is least important which is in accordance with the factor of safety equation for dry condition. However, its order of importance increases to 4th and 2nd place in regression model 3 and regression model 4 respectively.

5. REFERENCES

Chatterjee, S. and Price, B. (1977) "Regression analysis by example", John Willey & Sons, New York, pp. 228.

Davis, J.C. (1973) "Statistics and data analysis in geology", John Willey & sons, New York pp.550.

Draper, N.R. and Smith, H. (1966) "Applied Regression Analysis", John Willey & Sons, New York.

Hoek, E. and Bray, J.W. 1981 "Rock slope engineering" (3rd edition), Institution of Mining and Metallurgy, London, pp. 358.

Sharma, S. (1995) "Geotechnical Investigations of Lakhwar Dam, Garhwal Himalaya, India". Unpublished Ph.D. thesis, University of Roorkee, Roorkee, pp. 181.

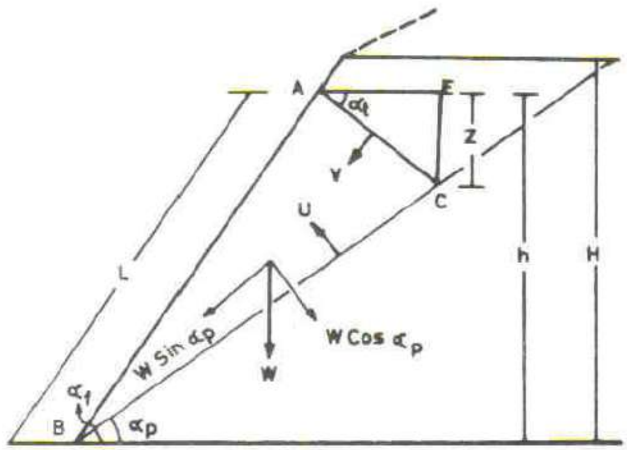

FIG. 1 GEOMETRY OF THE SLOPE

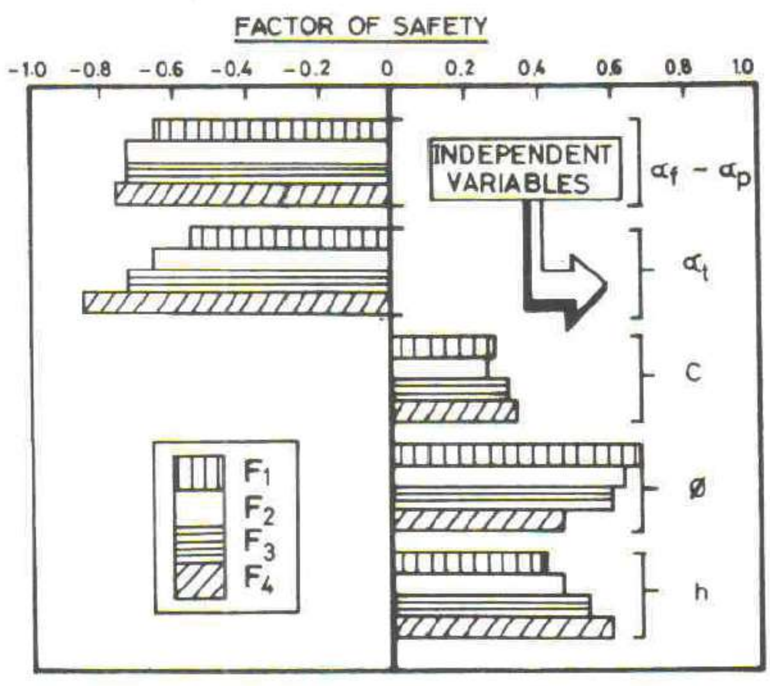

FIG. 2 CORRELATION MATRIX OF FACTOR OF SAFETY WITH INDEPENDENT VARIABLES

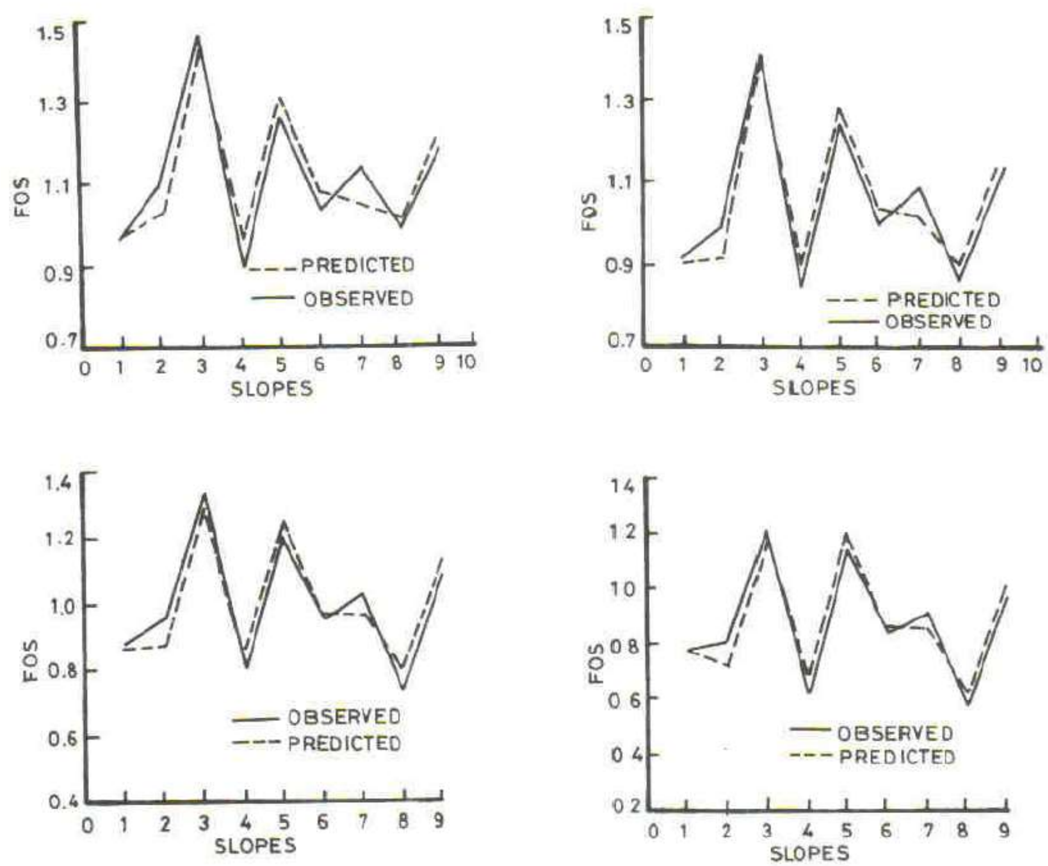

FIG. 3 COMPARISON OF OBSERVED AND PREDICTED FACTOR OF SAFETY.
A) REGRESSION MODEL 1, B) REGRESSION MODEL 2, C) REGRESSION MODEL 3, D) REGRESSION MODEL 4.

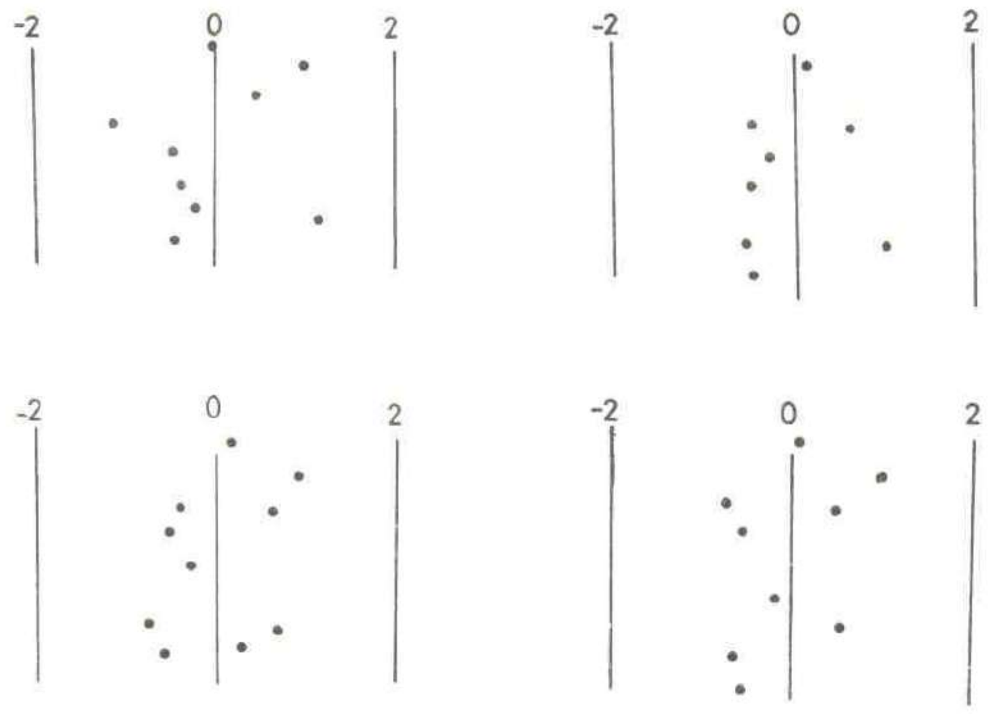

FIG 4. PLOT OF STANDARIZED RESIDUALS.
A) REGRESSION MODEL 1, B) REGRESSION MODEL 2,
C) REGRESSION MODEL 3, D) REGRESSION MODEL 4.

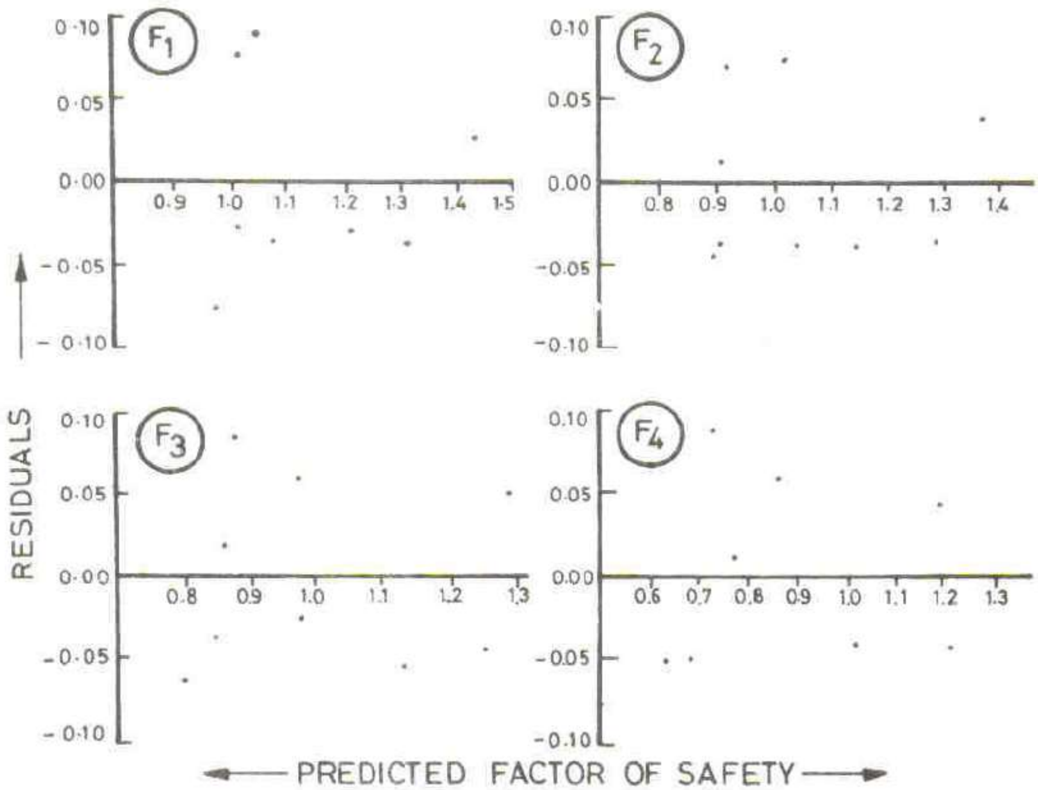

FIG. 5 PLOT OF RESIDUALS AGAINST PREDICTED FACTOR OF SAFETY FOR ALL MODELS