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Polyaxial Compressive Strength of Concrete Cubes Representing Rock

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ABSTRACT

The deep underground structures and deeply buried facilities in rocks are subjected to high confining pressure and failure takes place under the influence of multiaxial state of stress. The compressive strength obtained from conventional rock triaxial testing cannot predict the actual strength behaviour when the rock is under polyaxial state of stress. A solution based on the polyaxial state of stress conditions is very much required. The present study is an attempt toward understanding strength behaviour of rocks under polyaxial stress condition. Polyaxial compressive strength of concrete cube representing rock specimen (hereinafter referred as model rock) was determined by conducting tests in polyaxial rock testing machine. Concrete of M20 grade was used to represent model of rock material. The rock specimens having dimensions of 100mm x 100mm x 100mm were loaded to failure under various combinations of intermediate and minor principal stresses. The minor principal stress, σ_3 was varied from 2.5 to 10 MPa and the intermediate principal stress, σ_2 varied from 2.5 to 30 MPa. The major principal stress, σ_1 was increased at a rate of 1MPa/min until failure occurred. It is observed that the intermediate principal stress has substantial effect on strength of such modelled rock. Applicability of various polyaxial strength criteria in vogue has been assessed by comparing error in prediction of polyaxial strength. It was observed that the least error in prediction was shown by modified Mohr-Coulomb criterion followed by modified Weibols and Cook criterion.

Keywords: Polyaxial state; Model rock; Modified Mohr-Coulomb criterion

1. INTRODUCTION

Civil Engineering structures situated in rocks are generally subjected to polyaxial stress conditions where all the principal stresses have different magnitudes. While analyzing strength behaviour of rocks, it is customary to ignore the effect of intermediate principal stress (σ_2) on the strength of the rock. As a consequence the strength is represented as a function of minor principal stress (σ_3) only. During past few decades, it has been realized that the intermediate principal stress (σ_2) has substantial effect on strength of rock. A few attempts have been made by researchers to investigate the effect of intermediate principal stress on strength of rock. Some of the prominent studies are by Murrell (1963), Handin et al. (1967), Mogi (1967; 1971), Michelis (1985), Takahashi and Koide (1989), Wawersik et al. (1997), Haimson and Chang (2000), Kwaśniewski and Takahashi (2007), Cai (2008) and

Sriapai et al. (2013). It is felt that these studies are inadequate and attempts are being made to conduct more laboratory studies on polyaxial strength of rocks. The major difficulty faced in laboratory is that polyaxial strength test is very difficult and the facility to conduct such tests is available at only few places in the world. Recently polyaxial test facility has been acquired at IIT Roorkee and systematic studies are being planned and conducted to understand the polyaxial strength behaviour of rocks. The present paper discusses in detail an experimental study, wherein a concrete cube representing rock specimen (hereinafter known referred as model rock) has been tested under polyaxial stress condition. Substantial effect of intermediate principal stress on strength of model rock has been observed. Applicability of polyaxial strength criteria in vogue has been evaluated in the light of test results.

1.1 Earlier Studies on Polyaxial Strength

The effect of intermediate principal stress on the strength of intact rocks was first observed by Murrell (1963). He carried out triaxial extension and compression tests on Carrara marble. He observed that for any given value of minor principal stress, the strength was larger in triaxial extension than in triaxial compression indicating that there was an effect of intermediate principal stress. Similar behaviour was reported by Handin et al. (1967) while conducting tests on Solenhofen limestone, Blair dolomite and Pyrex glass. Mogi (1967, 1971 and 2006) studied rocks under polyaxial compression conditions using a self-designed polyaxial compression apparatus. Prismatic specimens of three different rock types were used in the study. Michelis (1985) also developed polyaxial testing equipment and studied the behaviour of prismatic specimens of dense marble under various stress conditions. Takahashi and Koide (1989) built a modified version of Mogi's apparatus and tested samples of three different rock types. Haimson and Chang (2000) built another, highly sophisticated version of a Mogi type apparatus and studied the behavior of KTB amphibolite and Westerly granite under high σ_2 and high σ_3 conditions (Chang and Haimson, 2000, 2005). Cai (2008) used numerical technique to explain the effect of intermediate principal stress on strength of rock. Li et al. (2012) summarized various polyaxial testing equipment developed throughout the world and classified them into three categories namely Type-I (Rigid platen type), Type –II (Flexible medium type) and Type –III (Mixed type). Sriapai et al. (2013) conducted polyaxial tests on Maha Sarakham salt with the help of indigenously developed polyaxial testing frame which apply lateral load with the help of cantilever beam system.

Various strength criteria were proposed by researchers to predict the strength of intact rock when subjected to polyaxial state of stress. Colmenares and Zoback (2002), Al-Ajmi and Zimmerman (2005), Benz and Schwab (2008) and Singh et al. (2011) statistically examined different failure criteria by applying them to the published polyaxial test data. A comprehensive summary on the polyaxial strength criteria has been presented by Singh et al. (2011). They suggested a modification to simple Mohr-Coulomb criterion by imbibing critical state concept and proposed modified Mohr-Coulomb criterion for triaxial and polyaxial strength of rocks and model rock mass also.

It can be seen that the laboratory studies on polyaxial strength of rock are very few and hence there is a need for larger database. With this in mind the following experimental program was conducted.

2. EXPERIMENTAL PROGRAM

An experimental study has been carried out to determine the effect of intermediate principal stress on the strength of a model rock. The complete procedure involved the preparation of model rock specimen and testing is discussed below.

2.1 Model Material

Model materials are commonly used to simulate the behaviour of rocks as they show less scatter and significant conclusion can be drown out from experimental observations. In the present study concrete has been used as a model rock. Ordinary Portland cement conforming to Grade 43 has been used as binder. Locally available river sand having fineness modulus in the range of 2.6 to 2.9 was used as fine aggregate. Aggregate in its natural form (uncrushed river gravel) having maximum size of 12 mm was used as coarse aggregate. The 28 day nominal cube strength of 20 MPa was designed using the absolute volume method as per IS 10262-2009. The proportion corresponding to Table 1 was used as mix design. The constituent materials were mixed in the laboratory manually and cubes were cast in steel moulds which were vibrated well to ensure uniformity. The specimens were cured for 28 days and then dried in air for 15 days.

Twenty five cubical specimens of 100mm size were prepared and used for polyaxial and uniaxial tests. The specimens were carefully grounded and lapped on all surface and their orthogonality was maintained. Figure 1 shows the specimens used for the present study.

Table 1: Mix proportion for concrete

Cement (kg/m^3)	Fine Aggregate (kg/m^3)	Coarse Aggregate (kg/m^3)	Water (l/m^3)	Cube Compressive Strength (N/mm^2)	Aggregate Size (mm)
380	893	932	209	24.9	12 mm passing

Figure 1: Cubical specimens used for present study

2.2 Testing Apparatus

The tests were conducted in Geotechnical Engineering Laboratory at IIT Roorkee. Polyaxial rock testing system developed by Wille-Geotechnik, Germany had been used for conducting the polyaxial tests on cubical specimens of the model rock. The machine as shown in Fig. 2 consists of power-pack, three pressure intensifiers and the specimen holder. The power-pack generates the initial pressure which gets intensified in the intensifiers. The specimen holder can accommodate specimen of size 100mm x 100mm x 100mm. The compressive stresses are applied on the specimen with six pressure plates, two pressure plates for each direction. The pressure plates have dimensions of 94mm x 94mm. The intensified pressure moves the pressure plates which create the stresses in the specimen from all three directions. The maximum value of principal stress which can be attained in this machine is 140 MPa. The deformation in the specimen was measured with the LVDT attached with the pressure plates. The test automatically stops when the cumulative deformation in loading direction become equal to 4mm. The complete system is fully automated and controlled by GEOsys software which controls test appliances that carry out and coordinate various test operations.

Figure 2: Polyaxial testing machine at IIT Roorkee

2.3 Test Programme

Table 2 shows the test programme carried out for the present study. The polyaxial tests were conducted on 20 specimens. The tests have been conducted at four different minor principal stresses (σ_3) with values of 2.5, 5, 7.5 and 10MPa respectively. The intermediate principal stress (σ_2) was kept such that the ratio of intermediate to minor principal stress was equal to 1, 1.5, 2, 2.5 and 3 respectively. The uniaxial compressive strength (UCS) tests were conducted on five cubical specimens in another compression testing machine. Teflon sheets of 0.5mm thickness were used to reduce the friction between loading plates and specimen during UCS and polyaxial strength tests.

Table 2: Test program for the present study

Specimen Number	σ_3 (MPa)	σ_2 (MPa)	σ_2/σ_3	Specimen Number	σ_3 (MPa)	σ_2 (MPa)	σ_2/σ_3
1	0	0	-	12	7.5	7.5	1
2	2.5	2.5	1	13	7.5	11.25	1.5
3	2.5	3.75	1.5	14	7.5	15.0	2
4	2.5	5.0	2	15	7.5	18.75	2.5
5	2.5	6.25	2.5	16	7.5	22.5	3
6	2.5	7.5	3	17	10.0	10.0	1
7	5.0	5.0	1	18	10.0	15.0	1.5
8	5.0	7.5	1.5	19	10.0	20.0	2
9	5.0	10.0	2	20	10.0	25.0	2.5
10	5.0	12.5	2.5	21	10.0	30.0	3
11	5.0	15.0	3				

2.4 Testing Procedure

The model rock specimen was placed in the sample holder and the pressure plates were touched to it. An initial hydrostatic pressure of 1MPa was applied in all three directions as initial seating pressure. This initial seating pressure is maintained for some time to make sure that the loading plates are in proper contact of specimen from all the three directions. The pressures in all the three directions are then hydrostatically increased at a constant rate of 1MPa/min till all round pressure equal to σ_3 is reached. Now σ_3 is maintained at constant level and σ_2 and σ_1 are increased till both of them reach the specified σ_2 value. Now σ_2 is also maintained at constant level and σ_1 is increased till failure. The pressure from direction of intermediate and minor principal stress will automatically maintain its level once it reached the specified value. The failure of the specimen was indicated by shooting up of strain in the direction of major principal stress. Once the failure of specimen is observed, the testing is stopped and specimen was carefully taken out from the specimen holder to note down the failure mode. Figure 3 shows the typical pressure v/s time and deformation v/s time plots for the testing.

3. EXPERIMENTAL RESULTS

3.1 Characterization of Rock Material

Cylindrical cores having diameter of 54mm were drilled from concrete cubes for characterization of the model material. The static cum dynamic-rock-triaxial testing equipment at IIT Roorkee was used to conduct the UCS and triaxial tests. The average UCS of the five cylindrical specimens was 21.04MPa while for the cubical specimen it was 24.23 MPa. The average tensile strength of rock obtained from Brazilian tests is 2.3 MPa. The average tangent modulus of rock specimen is 6.0 GPa. The rock can be classified as EM as per Deere Miller Classification (1966).

The triaxial tests on cylindrical specimens were conducted at five different confining pressure of σ_3 = 0, 2.5, 5, 7.5, 10 MPa respectively. The Mohr-Coulomb failure envelope obtained from triaxial tests is shown in Fig. 4. The values of cohesion and internal friction for the range of σ_3 values mentioned above are 5.5 MPa and 39.56° respectively. The physical and engineering properties of the model material are presented in Table 3.

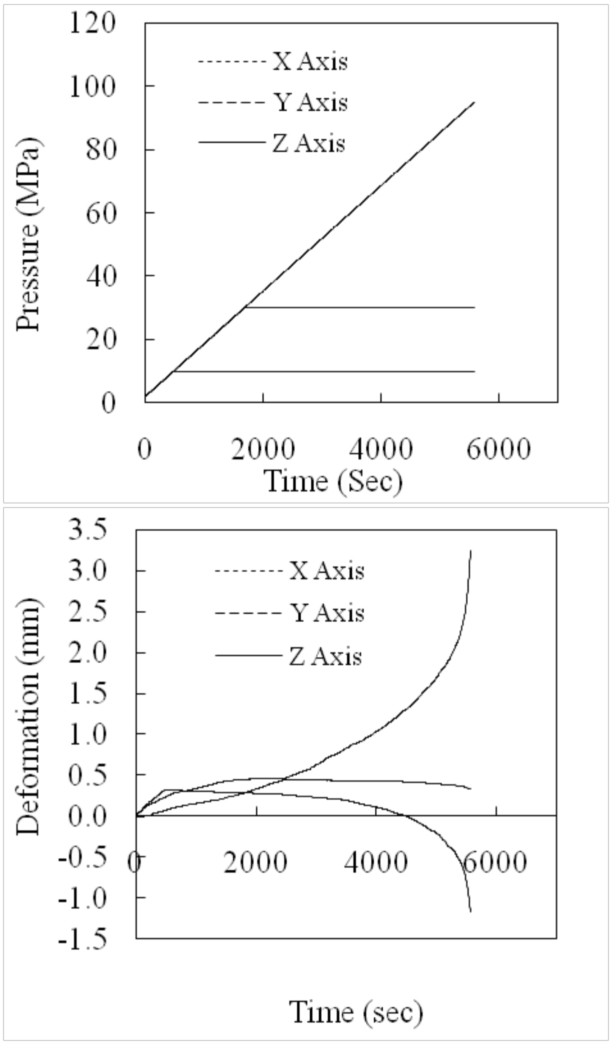

Figure 3: Typical pressure v/s time and deformation v/s time plot

Table 3: Properties of the model material (Based on cylindrical specimens)

Property	Symbol	Value
Unit weight (kN/m^3)	γ	24.1
Uniaxial compressive strength (MPa)	σ_{ci}	21.04
Tensile strength (MPa)	σ_t	2.3
Tangent modulus (GPa)	E_{t50}	6.0
Cohesion (MPa)(for σ_3 = 0 - 10 MPa)	c_i	5.5
Friction angle (°) (for σ_3 = 0 - 10 MPa)	φ_i	39.56
Deere-Miller classification (1966)	-	EM

3.2 Failure Modes in Polyaxial Tests

Figure 5 shows few typical failed specimens after polyaxial tests. It was observed that a number of cracks had developed throughout the specimens mostly developing from edge of the specimen. The cracks propagate through the interface of the aggregate and mortar. The fracture plane is observed to have strike almost aligned with the plane of intermediate failure stress (σ_2). The dip of the fracture plane is oriented toward the direction of minor principal stress (σ_3).

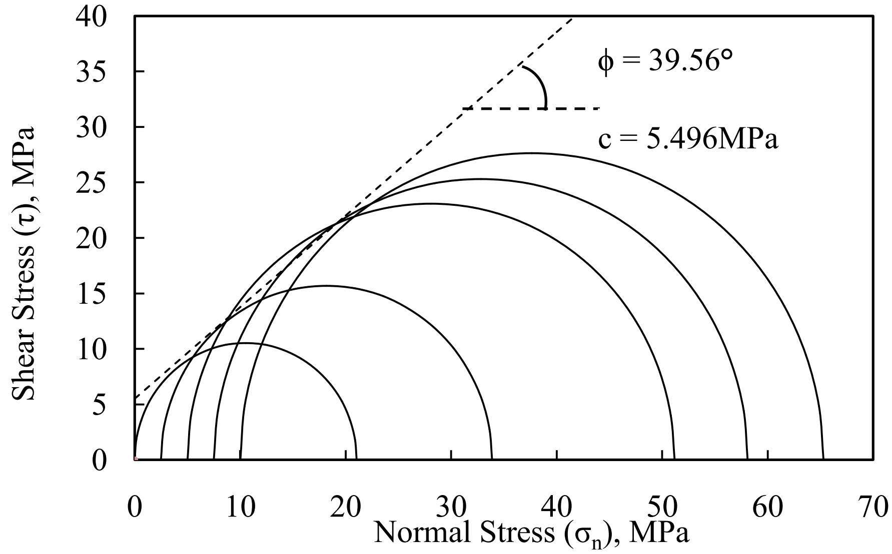

Figure 4: Failure envelope from triaxial tests (cylindrical specimens)

3.3 Stress-Strain Plots

Few typical stress-strain plots from polyaxial strength tests are presented in Fig. 6. Deformation measured in respective direction was used to calculate principal strain (ε_1, ε_2, ε_3) in that direction. The volumetric strain (ε_v) was calculated as:

$$\varepsilon_v = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \tag{1}$$

The peak value of the major principal stress was considered as the strength and the results in term of the principal stresses (σ_1, σ_2, σ_3) at failure are presented in Table 4. It is observed that the strength of rock substantially depends upon the intermediate principal stress and its value increases with increase in intermediate principal stress. Figure 7 shows the failure stress as a function of intermediate failure stress for different σ_3 values.

Table 4: Results of polyaxial testing (all values in MPa)

Specimen No.	σ_3	σ_2	σ_1	Specimen No.	σ_3	σ_2	σ_1
1	0	0	24.23	12	7.5	7.5	59.33
2	2.5	2.5	35.74	13	7.5	11.25	65.08
3	2.5	3.75	37.62	14	7.5	15	56.67
4	2.5	5.0	38.65	15	7.5	18.75	60.67
5	2.5	6.25	39.77	16	7.5	22.5	70.11
6	2.5	7.5	43.00	17	10	10	73.82
7	5	5	47.92	18	10	15	74.06
8	5	7.5	50.66	19	10	20	72.33
9	5	10	54.28	20	10	25	73.93
10	5	12.5	56.16	21	10	30	89.91
11	5	15	48.43				

Figure 5: Typical failed cubical synthetic rock specimen

4. STRENGTH CRITERIA FOR POLYAXIAL STRENGTH

Strength criteria are used to predict the strength of rock for any state of stress. Parameters of the strength criterion are obtained through conventional triaxial tests at low confining pressure. A strength criterion can be said to be robust, if it can predict the strength under polyaxial condition using the parameters from triaxial tests conducted at low confining pressure condition. Seven failure criteria were selected and their predictive capabilities in respect to the database generated in present study have been evaluated. To obtain criterion parameters, first three data points from polyaxial tests corresponding to UCS and first two triaxial condition (i.e. $\sigma_2 = \sigma_3$) were used. Results of triaxial tests on cylindrical specimens were not used as effect of shape (cylindrical vs. cubical) and size will have additional influence. Since the aim of present analysis is to compare various criteria, triaxial stress data from polyaxial tests on cubes were considered to get the criterion parameters. The data used for obtaining the parameters and corresponding Mohr-Coulomb parameters are given in Table 5. Table 6 shows the resulting parameters for seven criteria obtained from first three triaxial data points. Strength values were predicted by using all the criteria. The predicted values from different criteria have been compared with the experimental values. The predictive capability of the strength criterion is determined using an index i.e. average percent error (AVPE) value as an indicator. The lower AVPE value indicates a better predictability of the criterion. The average percent error (AVPE) for a data set is computed as:

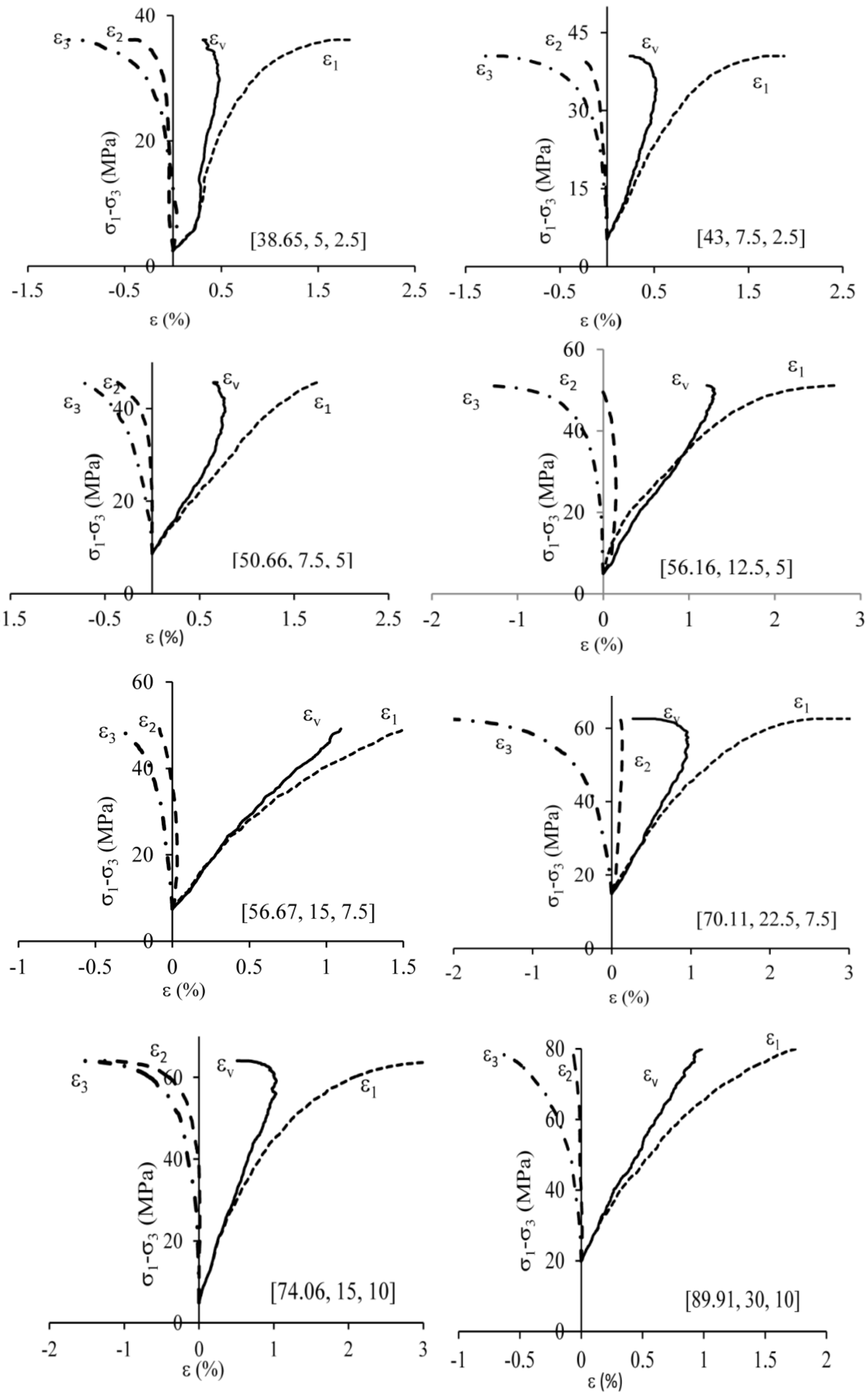

Figure 6: Typical stress-strain plot for polyaxial testing of model rock (value in parenthesis represent the values as σ_1, σ_2, $\sigma_3$in MPa)

$$\text{AVPE} = \sqrt{\frac{1}{\text{npt}}\sum_{i=1}^{\text{npt}} \text{pe}^2} \qquad (2)$$

where *npt* is the number of data points, pe the percent error in prediction for a data point and was computed as:

$$pe = abs\left(\frac{\sigma_{1cal} - \sigma_{1exp}}{\sigma_{1exp}} x100\right)\% \tag{3}$$

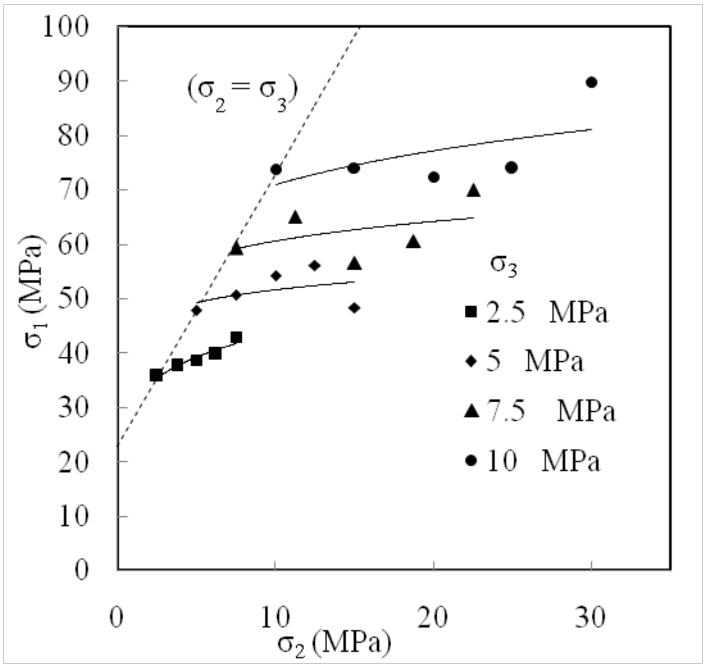

Figure 7: Strength results in term of major principal stress as a function of intermediate principal stress

where σ_{1exp} and σ_{1cal} are experimental and predicted values of the strength of the model rock under polyaxial condition.

The seven criteria used for the study were modified Wiebols and Cook criterion (Zhou, 1994), Drucker-Prager Criterion (Drucker and Prager, 1952), Modified Lade and Duncan criterion (Ewy, 1999), Mogi-Coulomb criterion (Al-Ajmi and Zimmerman, 2005), extended Mogi-Coulomb criterion (Al-Ajmi, 2006), modified Mohr-Coulomb criterion (Singh et al., 2011). A brief description of all these criteria is given in Annexure I. The percent error in prediction was calculated for each data point and AVPE was calculated for model rock for different criteria.

Table 5: Polyaxial test data used for calibration of the model parameters (All Values in MPa)

σ_3	σ_2	σ_1
0	0	24.23
2.5	2.5	35.74
5	5	47.92
Mohr-Coulomb Parameters C = 5.54, φ = 40.65°		

Table 6: Parameters calibrated for the different strength criteria

Strength criterion	Criterion parameters
Modified Wiebols and Cook (Eq. A.1)	σ_3 = 0.0MPa: A = 4.13MPa, B =1.47 C= -0.03 MPa^{-1} σ_3 = 2.5MPa:A = 3.61MPa,B =1.47 C= -0.02MPa^{-1} σ_3 = 5.0MPa:A = 3.25MPa,B=1.48 C= -0.02MPa^{-1} σ_3 = 7.5MPa:A = 2.99Mpa,B= 1.49 C= -0.02MPa^{-1} σ_3 = 10.0MPa:A = 2.79MPa,B =1.50 C= -0.01 MPa^{-1}
Circumscribed Drucker- Prager (Eq. A.9)	α= 0.96 k = 6.2
Inscribed Drucker- Prager (Eq. A.9)	α =0.61 k =3.93
Modified Lade (Eq. A.16)	S = 6.45, η = 37.58
Mogi-Coulomb (Eq. A.21)	a = 3.96, b = 0.61
Extended Mogi- Coulomb (Eq. A.28)	a = 4.3, b = 0.58, c = 0.001
Modified Mohr- Coulomb (Eq. A.29)	c = 5.54, ϕ =40.66, σ_{ci} = 24.12

A summary of outcome of the analysis has been given in Table 7, wherein the AVPE values obtained through different criteria has been shown. The Fig. 8 shows the AVPE values for different criteria. It can be seen that modified Mohr-Coulomb criterion gives the least error followed by modified Weibols and Cook criterion for the data base used in this study.

Table 7: Average percent error for different criteria

Criterion	AVPE (%)
Modified Weibols and Cook	9.77
Circumscribed Drucker-Prager	22.05
Inscribed Drucker-Prager	43.48
Modified Lade	16.3
Mogi-Coulomb	10.78
Extended Mogi-Coulomb	15.81
Modified Mohr-Coulomb	8.06

5. PROBABILITY OF PREDICTION

During analysis and design, the designer will be interested in knowing the confidence level if a particular criterion is used for analysis. There will always be some error in predicting the strength values. A criterion which has higher probability of predicting strength value within specified permissible error will carry more confidence. An exercise was done using the present database and cumulative distribution curves were obtained for the percent error (pe) computed for each criterion. The cumulative distribution curves are shown in Fig. 9 for all the criteria. If a point is selected on the probability distribution curve with coordinates as X and Y, then there will be a probability Y of predicting strength such that error will be less than or equal to X. If permissible error in prediction is considered 20% then the Fig. 9 shows that modified Mohr-Coulomb criterion has about 95% probability that error in prediction will be less than or equal to 20%. Similarly for modified Weibols and Cook criterion the probability will be about 88%. The others criteria will have quite low probability of predicting error within 20%. The above analysis indicates that higher degree of confidence can be expected in simple modified Mohr-Coulomb for strength prediction.

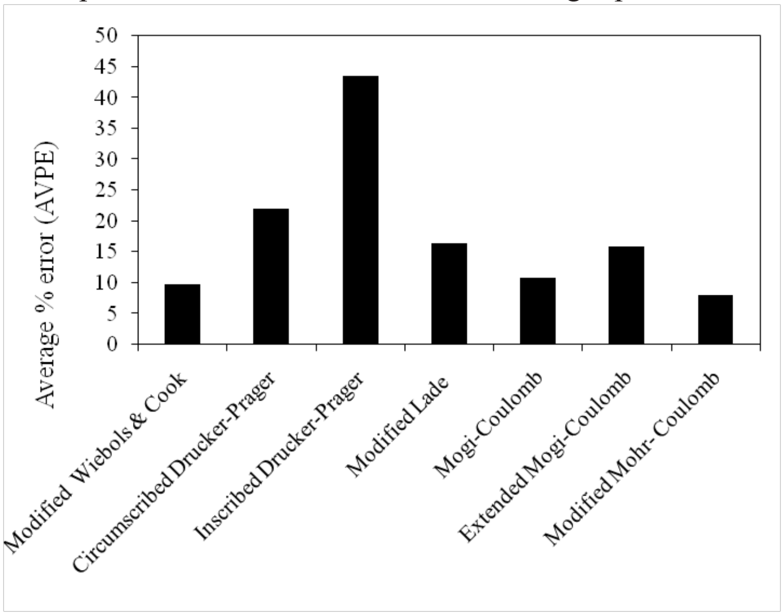

Figure 8: Comparison of average percent error due to different polyaxial strength criteria

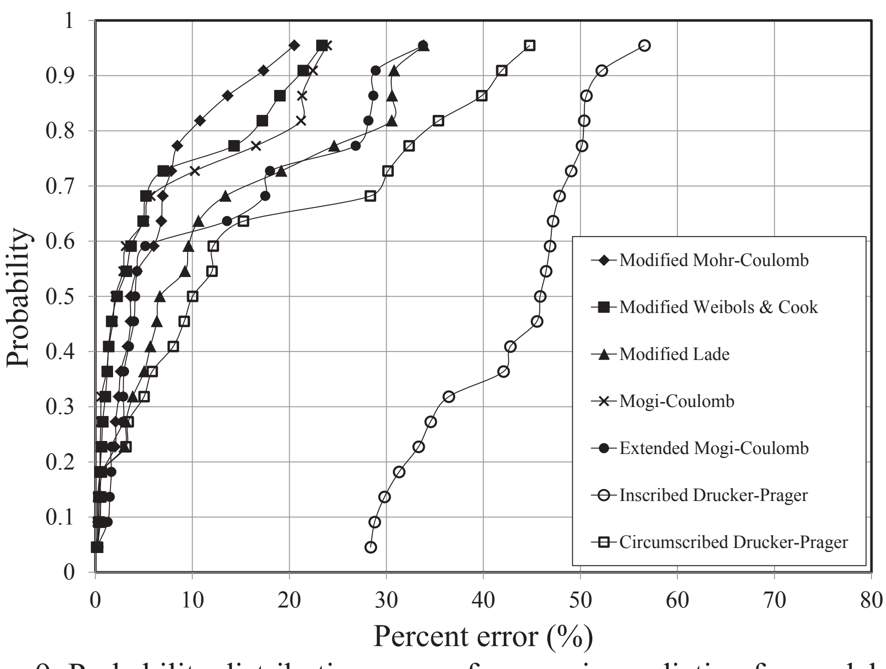

Figure 9: Probability distribution curves for error in prediction for model rock

6. CONCLUDING REMARKS

The objective of the present experimental work was to analyze the effect of intermediate principal stress on the strength of model rock under polyaxial state of stress. An experimental investigation was carried out where a model rock of M20 concrete was subjected to polyaxial stresses such that the minor principal stress was kept in range of 2.5-10 MPa while the intermediate principal stress was kept in the range of 2.5-30 MPa. Substantial effect of intermediate principal stress on strength of the rock has been observed. Failed specimens show that the shear fractures strike in the direction of intermediate principal stress and dip along minor principal stress direction.

A comparative analysis of different polyaxial criteria in vogue has been carried out using the database obtained from the experimental program. Average percent error (AVPE) has been used as an indicator of the predictive capability of the seven criteria considered. Among all the criteria, modified Mohr-Coulomb criterion predicted better results followed by the similar modified Weibols and Cook criterion for the database used in this study. The study also indicates that for simple modified Mohr-Coulomb criterion (Eq. A.29), there will be 95% probability that the error in prediction will be within 20%, which is acceptable. The above non-linear and simple criterion is also applicable to concrete technology and so concrete linings.

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Annexure I

1. Modified Weibols-Cook Criterion

The Modified Wiebols and Cook criterion based on the additional energy stored around Griffith cracks is an extension of the Drucker –Prager criterion. The failure criterion described by Zhou (1994) defines J_2 at failure in terms of J_1 as:

$$J_2^{1/2} = A + BJ_1 + CJ_1^2 \qquad \text{A.1}$$

Where,

$$J_1 = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \qquad \text{A.2}$$

$$J_2^{1/2} = \sqrt{\frac{1}{6}\left((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right)} \qquad \text{A.3}$$

The parameters *A, B* and *C* are determined under triaxial condition, such that

$$C = \frac{\sqrt{27}}{2C_1 + (q-1)\sigma_3 - C_o}\left(\frac{C_1 + (q-1)\sigma_3 - C_o}{2C_1 + (2q+1)\sigma_3 - C_o} - \frac{q-1}{q+2}\right) \qquad \text{A.4}$$

$$C_1 = (1 + 0.6\mu_i)C_0 \qquad \text{A.5}$$

C_0 = UCS of the rock

$$q = \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right) \qquad \text{A.6}$$

$$B = \frac{\sqrt{3}(q-1)}{q+2} - \frac{C}{3}\left(2C_o + (q+2)\sigma_3\right) \qquad \text{A.7}$$

$$A = \frac{C_0}{\sqrt{3}} - \frac{C_0}{3}B - \frac{C_0^2}{9}C \qquad \text{A.8}$$

2. Drucker Prager criterion

The extended Von Mises yield criterion or Drucker-Prager criterion was originally developed for soil mechanics (Drucker and Prager, 1952). It can be expressed as

$$J_2^{1/2} = k + \alpha J_1, \qquad \text{A.9}$$

Where

$$J_1 = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \qquad \text{A.10}$$

and

$$J_2^{1/2} = \sqrt{\frac{1}{6}\left((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right)} \qquad \text{A.11}$$

Where α and k are material constants and are related to the internal friction and cohesion of the material. The two versions of the Drucker-Prager criterion are:

The inscribed Drucker-Prager criterion:

$$\alpha = \frac{3\,\mathrm{Sin}\,\phi}{\sqrt{9 + 3\,\mathrm{Sin}^2\,\phi}} \qquad \text{A.12}$$

and

$$k = \frac{3C_0 \cos\phi}{2\sqrt{q}\sqrt{9 + 3\,\mathrm{Sin}^2\,\phi}} \qquad \text{A.13}$$

The circumscribed Drucker-Prager criterion:

$$\alpha = \frac{6\,\mathrm{Sin}\,\phi}{\sqrt{3}\,(3 - \sin\phi)} \qquad \text{A.14}$$

and

$$k = \frac{\sqrt{3}\,C_0 \cos\phi}{\sqrt{q}\,(3 - \sin\phi)} \qquad \text{A.15}$$

Where ϕ is the angle of internal friction.

3. Modified Lade and Duncan Criterion

The Lade criterion is a three-dimensional failure criterion for friction materials without effective cohesion. The criterion according to Ewy (1999) is

$$\frac{I_1^3}{I_3} = (3^3 + \eta) \qquad \text{A.16}$$

where the first and third stress invariants can be written as

$$I_1 = (\sigma_1 + S) + (\sigma_2 + S) + (\sigma_3 + S) \qquad \text{A.17}$$

and

$$I_3 = (\sigma_1 + S)(\sigma_2 + S)(\sigma_3 + S) \qquad \text{A.18}$$

The parameter S and η are

$$S = \frac{c}{\tan\phi} \qquad \text{A.19}$$

and

$$\eta = \frac{4\tan^2\phi(9-7\sin\phi)}{1-\sin\phi} \qquad \text{A.20}$$

4. Mogi-Coulomb Criterion

Al-Ajmi and Zimmerman (2005) proposed a new failure criterion known as Mogi Coulomb criterion.

$$\tau_{oct} = a + b\,\sigma_{m,2} \qquad \text{A.21}$$

where

$$\tau_{oct} = \frac{1}{3}\left((\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2 \right)^{1/2} \qquad \text{A.22}$$

$$a = \frac{2\sqrt{2}}{3}\frac{\sigma_{ci}}{k+1} \qquad \text{A.23}$$

$$b = \frac{2\sqrt{2}}{3}\frac{k-1}{k+1} \qquad \text{A.24}$$

$$\sigma_{m,2} = \frac{\sigma_1+\sigma_3}{2} \qquad \text{A.25}$$

$$\sigma_{ci} = \frac{2c\sin\phi}{1-\sin\phi} \qquad \text{A.26}$$

$$k = \frac{1+\sin\phi}{1-\sin\phi} \qquad \text{A.27}$$

5. Extended Mogi-Coulomb criterion

Al-Ajmi (2006) formulated a new criterion by extending the Mogi-Coulomb criterion and introduced a nonlinear or parabolic Mogi criterion given as:

$$\tau_{oct} = a + b\sigma_{m,2} + c\sigma_{m,2}^2 \qquad \text{A.28}$$

The parameters a and b represent the cohesion and angle of internal friction. The parameter c is a curve fitting parameter that represents the non-linear behaviour at high effective mean stresses.

6. Modified Mohr-Coulomb Criterion

Singh et al. (2011) made some modification to Mohr-Coulomb Criterion and proposed a new non-linear criterion for intact rock known as Modified Mohr-Coulomb (MMC) criterion. The criterion was based on critical stress concept given by Barton (1976). The criterion directly uses Mohr-Coulomb shear strength parameters (c_{i0} and φ_{i0}) which should be obtained by performing few conventional triaxial strength tests at low confining pressure ($\sigma_3 \rightarrow 0$).

$$\left(\sigma_1-\sigma_3\right)=\sigma_{ci}+\frac{2\sin\phi_{i0}}{1-\sin\phi_{i0}}\left(\frac{\sigma_2+\sigma_3}{2}\right)-\frac{1}{\sigma_{ci}}\frac{\sin\phi_{i0}}{1-\sin\phi_{i0}}\left(\frac{\sigma_2^2+\sigma_3^2}{2}\right),\quad 0\le\sigma_3\le\sigma_2\le\sigma_{ci} \qquad \text{A.29}$$

where, the value of φ_{i0} can be obtained from experimental triaxial test data as:

$$A_i=\frac{\sum\left(\sigma_1-\sigma_3-\sigma_{ci}\right)}{\sum\left(\sigma_3^2-2\sigma_{ci}\sigma_3\right)},\quad 0<\sigma_3\le\sigma_{ci} \qquad \text{A.30}$$

$$B_i=-2A_i\sigma_{ci} \qquad \text{A.31}$$

$$\text{Sin}\varphi_{i0}=\frac{B_i}{2+B_i} \qquad \text{A.32}$$

For value of σ_2 or σ_3 greater then σ_{ci}, its value in right hand side of the equation should be replaced by σ_{ci}.