A Technical Note on

Non-Linear Finite Element Study of Lined Circular Tunnel

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1.0 INTRODUCTION

When an opening is created in a pre-stressed medium, the existing stresses are redistributed and the excavation surface moves in to the opening. This inward movement of excavation surface, if continues unchecked for sufficient time after excavation, may cause loosening of rock-mass around the opening due to yielding and formation of plastic zone. A series of progressive events may then commence leading to rock-fall and other forms of instability. Such events are harmful and must be avoided as these may cause loss of life and property in addition to disruption of tunneling. The underground excavations are lined to prevent instability, to reduce flow of ground water in to the opening, and to improve aesthetics of the tunnel. But, the lining is expensive and adds to the cost of tunnel. Hoek and Brown (1980) suggest " …… The basic aim of any underground excavation design should be to utilize the rock itself as the principal structural material, causing as little disturbance as possible during the excavation process and adding as little as possible in the way of concrete and steel support." The New Austrian Tunneling Method (NATM) and Norwegian Method of Tunneling (NMT) have been designed to meet this basic aim.

The conventional form of rock support system requires more time to erect than the application of shotcrete. Thus, the rock-mass gets relatively little opportunity to loosen in the modern methods of tunneling. Besides, the shotcrete forms a ring around the opening and becomes an integral part of the rock-mass. The rock – lining systems deform together in the normal and the tangential directions. These aspects are missing in the conventional support system consisting of steel joist and ribs. In this paper, elastic – plastic response of a circular lined opening located in an isotropic ground is derived with or without compatibility of tangential displacement. Subsequently, this study adopts a jointed ground and the study is repeated.

2.0 STATE OF THE ART

The analytical solutions of underground opening are available for a few simple cases involving isotropic and homogeneous medium. More solutions exist for hydrostatic loading than for non-hydrostatic loading (Obert and Duvall, 1967; Hoek and Brown, 1980). The non-linear solutions also follow similar trend. Fritz (1984), Read (1986) and Florence and schwer (1978) derived solution of an unlined circular opening under hydrostatic loading, while Detourney (1986, 1988) considered non-hydrostatic loading. The solutions for lined openings are even fewer. An elastic solution for normal support pressure under hydrostatic loading is available. It should be noted that, by virtue of circular symmetry, the importance of tangential displacement compatibility vanishes under hydrostatic initial stress field.

More work is available for buried pipeline, which resemble in behavior with a lined tunnel. In the terminology of buried pipelines, a lining is classified as rigid or flexible in which its stiffness is evaluated relative to the medium stiffness. A lining is said to be flexible if it interacts with the medium in such a way that the pressure distribution in the lining and the corresponding deflected shape results in a negligible bending moment at all points. The non-dimensional flexibility ratio F and compressibility ratio C provide a means to quantify medium-liner properties in buried pipelines. Einstein and Schwartz (1979) define these ratios as follows,

$$F = \frac{1-\upsilon_l^2}{1-\upsilon_m^2} \cdot \frac{E_m}{E_l} \cdot \frac{R^3}{I_l} \tag{1}$$

$$C = \frac{1-\upsilon_l^2}{1-\upsilon_m^2} \cdot \frac{E_m}{E_l} \cdot \frac{R}{A_l} \tag{2}$$

where,

E	=	Modulus of Elasticity,
ν	=	Poisson ratio,
R	=	Radius of opening,
I	=	Moment of inertia of, and
A	=	Area of cross section.

The subscripts l and m in Eqs. 1 and 2 stand for lining and medium, respectively. It can be seen that the ratios C and F are concerned with the extensional (thrust) and flexural (bending moment) behavior of the liner-medium system. These may be used in the context of tunneling also. However, there are some differences between buried pipeline and tunnel. The lining of a tunnel is loaded through the failure of rock-mass (Daemen and Fairhurst, 1972) while a buried pipeline is loaded by the backfill. Rodriguez (1985) has studied elastic lining - medium interaction in circular tunnels based on the above concepts. A similar nonlinear study is not found in the available published literature.

3.0 NUMERICAL STUDY

The problem of a deep circular underground opening subjected to high vertical initial stress is solved in the present study by finite-infinite-interface element method. Kumar and Singh (1988) first employed this method in the solution of an elastic lined tunnel problem. The details of this research in which a non-linear problem is solved are as follows. The interface conditions are written as full bond in which both normal and tangential displacement compatibility are imposed while in a no bond case only the normal displacement compatibility is imposed.

Problem Description

Radius of circular opening R	= 200.0 cm
Initial stress ratio (Horizontal to Vertical)	= 0.5

Medium Description

Modulus of elasticity	=	20 000.00 kg/cm^2
Poisson ration	=	0.20
Cohesion	=	25 kg/cm^2
Angle of internal friction	=	30^o
Yield law	=	Mohr-Coulomb
Flow Rule	=	Associated

Lining Description

Modulus of elasticity	=	200 000.00 kg/cm^2
Poisson Ratio	=	0.15
Behavior	=	Elastic
Thickness	=	20.0 cm
Interface condition	=	Full bond or no bond

4.0 NUMERICAL MODEL

The numerical model of this problem is shown in Fig. 1. It contains 280 nodes and 90 elements. The rock-mass is described by 63 isoparametric 8-node quadratic finite elements. The lining is represented by nine isoparametric 8-node quadratic finite elements. The far field of the problem is modeled by nine 5-node parametric infinite elements with an inverse type far field decay characteristic. Nine 6-node interface elements are placed at the lining-tunnel interface. These elements can easily simulate full bond or no bond behavior. The separation at the interface is not permitted.

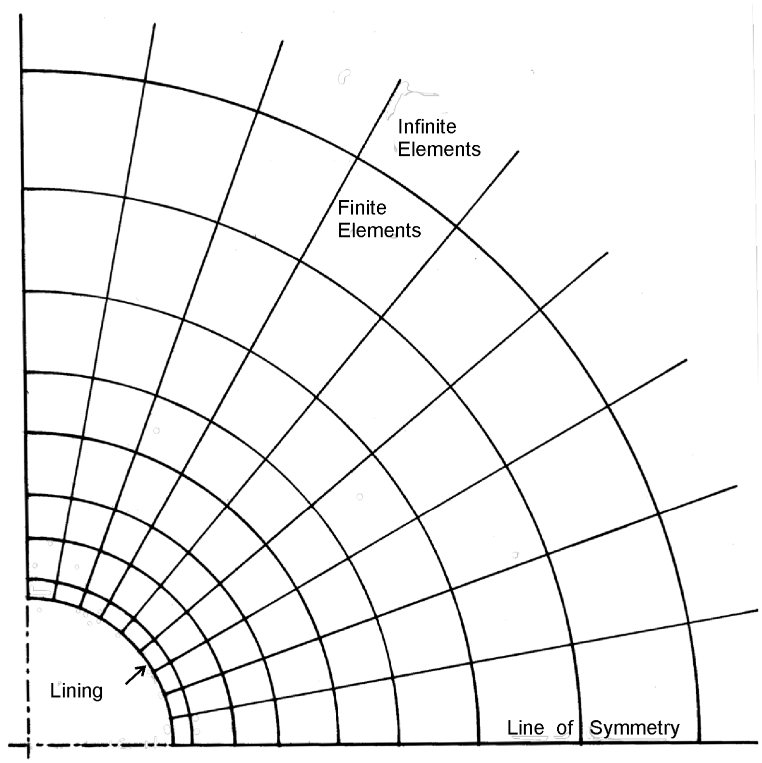

Fig. 1 - Finite infinite element model of lined tunnel

5.0 RESULTS OF ANALYSIS

Both linear and nonlinear analyses were performed in this study. The results of elastic study have already been published (Kumar and Singh, 1988) and are not repeated here. An elsto-viscoplastic solution technique is employed for nonlinear solution in which the steady state solution is taken as the final solution.

5.1 Non-Linear Solution in Isotropic Ground with Full Bond

It was found by trial and error that a combination of 24 MPa vertical and 12 MPa horizontal initial stresses introduced nonlinear effects in the numerical Model. The magnitudes of initial stresses were subsequently increased to 30 and 15 MPa respectively. The plastic region is under this arrangement is shown in Fig. 2, which also shows the distribution of normal and shear stress. The plastic zone in Fig. 2 forms at the shoulder of the opening. The analytical solutions of Detourney and John (1988) show that under non-hydrostatic loading, the plastic zone around unlined circular opening is butterfly shaped. This tendency is found in Fig. 2 also, even though an analytical solution for lined opening under non-hydrostatic loading is not available.

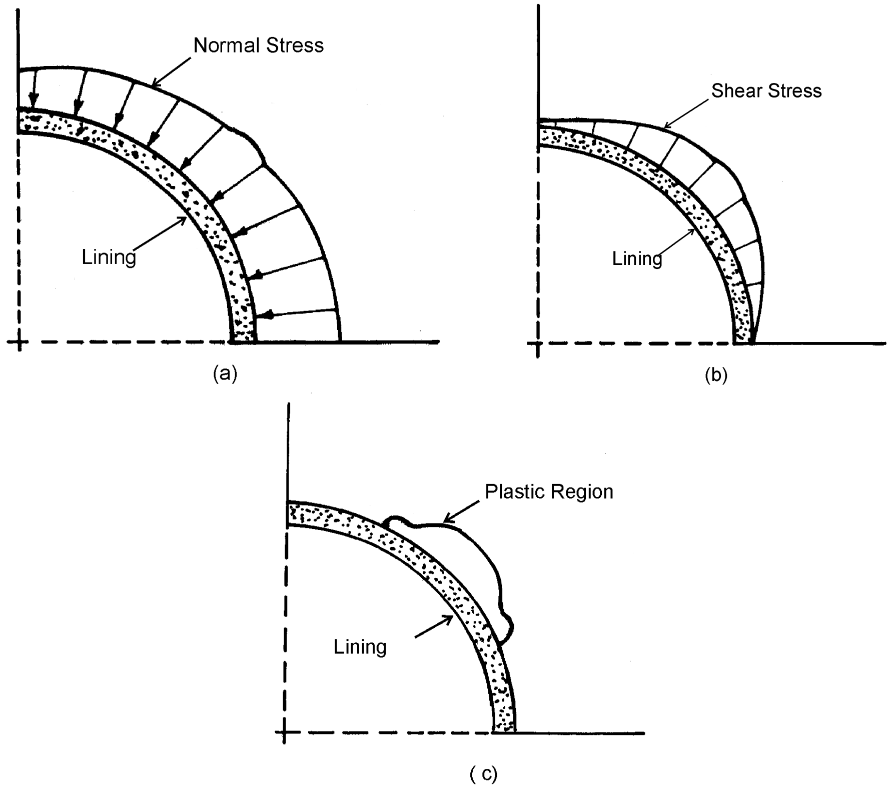

Fig. 2 - Results of analysis in isotropic ground

5.2 Non-Linear Solution in Isotropic Ground Under No Bond

The analysis is now repeated by reducing the tangential stiffness of the interface elements to zero. The solution did not converge even after 500 iterations and the plastic zone was still expanding when the solution was terminated.

5.3 Non-Linear Solution in Jointed Rock Mass with Full Bond

In this study, the jointed medium is converted to an equivalent continuum by the theory of Singh (1973). In a separate study (Kumar, 1999b), this numerical model was found to yield excellent results. Two joint orientations are considered in the present study and the results are presented in Fig. 3. It is seen that the location of plastic zone is now changed. The plastic zone now is localized and it may even be located away from the tunnel surface. It is not known which plastic zone formed first. This exercise demonstrates the effect of lining support on the failure zone but verification of these findings through alternative strategies is necessary.

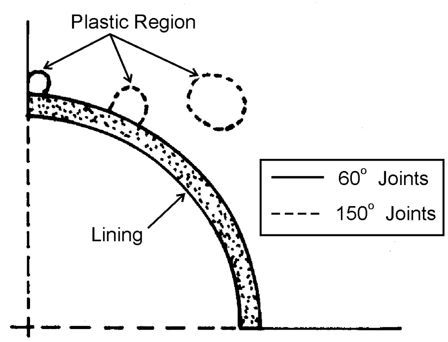

Fig. 3 - Results of analysis in jointed ground

5.4 Non-Linear Solution in Jointed Rock Mass with No Bond

The analysis is now repeated by reducing the tangential stiffness of the interface elements to zero. The solution did not converge even after 500 iterations and the plastic zone was still expanding when the solution was terminated.

6.0 CONCLUSION

This exercise demonstrates that the compatibility of tangential displacement at the rock-lining interface is of vital importance in containing the extent of plastic zone. Since shotcrete lining achieves this kind of compatibility over the conventional support system, it is very much likely that the efficiency of shotcrete support system and it successful use in numerous projects is due to the compatibility of tangential displacement at the rock-lining interface.

The size of broken zone constitutes loading on the lining. As is shown here, the size and location of broken zone change. The loading on the lining will also change correspondingly. Thus, due recognition to lining ground interaction is needed. In depth study of these concepts is needed which should address the effect of variation of opening size, lining thickness, medium properties and presence of ground surface.

Acknowledgement

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