Appropriate Loading Techniques in Finite Element Analysis of Mining Structures

J. Hematian
Apartment 504, 950
Warwick Street, Burlington
Ontario, Canada, L7T 325
E-mail: jhematian@yahoo.com

I. Porter[1]
Senior Lecturer
E-mail: i.porter@uow.edu.au
Tel: +61-2-42213451

R. N. Singh[1]
Professor & Discipline Leader
E-mail: raghu_singh@uow.edu.au
Tel: +61-2-42213070, Fax: +61-2-42213238

[1]*Division of Mining Engineering*
Faculty of Engineering, University of Wollongong
Northfields Avenue, Wollongong, NSW 2522
Australia

ABSTRACT

The application of numerical methods, such as the finite element method, to the geotechnical analysis of mining excavations is common practice. There is, however, some concern about the accuracy and validity of results obtained from these methods. Of all parameters which influence the results of FE analysis, loading technique is one of the most fundamental. The purpose of this paper is to illustrate the results obtained from similar models which are loaded in different ways. Key factors for choosing an appropriate loading method whilst considering the in-situ condition of the structure, will be addressed.

1. INTRODUCTION

The accuracy and validity of results obtained from finite element analysis is highly influenced by the parameters employed in the various stages of the modelling procedure. Analysis will usually produce some results which may or may not be correct or accurate. It has been mentioned in some reports that the magnitudes of deformation obtained from FE analysis were unrealistic compared with field data or that from theoretical solutions (Iannacchione 1990, and Hematian and Porter 1993). It is, therefore, very important to calibrate and optimise these parameters by preparing simple models that can have theoretical or obvious solutions. The results from FE analysis are then compared with the results from theoretical solutions.

Parameters used in a FE modelling procedure can be categorised in three major groups as follow:

i - Geometrical parameters: dimensions of the model, pattern and density of mesh.
ii - Structural parameters: element types and properties, freedom condition of grid points, boundary condition of different parts within the model and the model itself, and loading technique.
iii - Material parameters: constant values and/or constitutive equations that explain the behaviour of materials in different conditions.

In this research, 12 models were constructed and analysed to determine the most appropriate loading techniques for different conditions, which will result in the most accurate output. During this research, the 3-D finite element code, NASTRAN, was utilised. This program is a general purpose 3-D FE code for static and dynamic displacement and stress analysis of structures, solids and fluid systems (MSC/NASTRAN, 1991). NASTRAN can be employed to perform linear and non-linear analysis. The non-linear solutions consider both geometrical and material non-linearity. It executes the model with specific material properties under increasing load increments.

2. LOADING TECHNIQUES

There are two major methods to simulate the insitu stress state in the model. The first method is to consider gravitational load throughout the model. The second method is applying an equivalent force or stress at grid points or on the free faces of the model.

Since it is not practical to take into account the total cover of the structure in the FE model, it will be inevitable to use an apparent value for either the density of rocks or the gravity force. In either cases the gradient of stress in the model will be equal to H/h where H and h are depth of overburden and thickness of the model, respectively. This ratio is usually more than 10 when modelling underground structures at depths more than 300 m. Although inertial loading is an accurate method for modelling surface or shallow structures, application of this method for modelling deep structures will not provide a constant uniform stress state around the model (Figure 1).

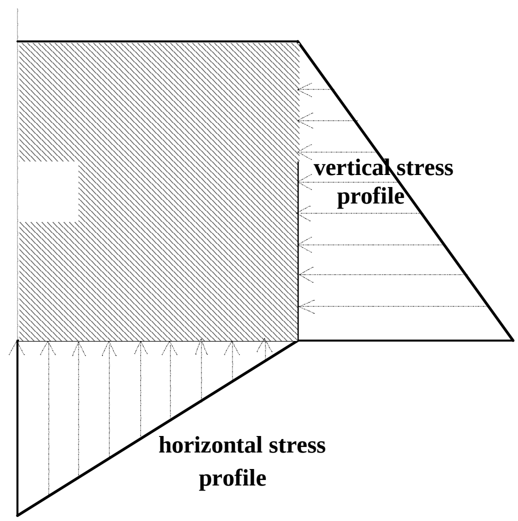

Figure 1- Stress profile around a model using inertial loading

The second method of simulating the in-situ stress state is to apply the same value of in-situ stresses on the free faces of the model or to calculate the equivalent forces and apply that at grid points around the model. In both cases, any effect resulting from the high stress gradient along the sides of the model will be eliminated. In this way, a constant uniform stress state will be achieved. The key point in this method is that there are differences between applying the load on the external or on the internal boundary of the model (Figures 2a and 2b).

Three series of models were constructed and analysed under four different loading conditions (σ_v = 10 MPa and the horizontal to vertical stress ratio, K = 1, 2, 3 and 4) to study the differences. The first group were virgin models without any opening and the stress state was applied on the external boundary. The second group were models including the structure, and stresses were applied on the external boundary. The third group of models were the same as the second group but stresses were applied on the internal boundary. These models are called Null, External and Internal Models, respectively. Figure 3 shows locations at which results were obtained.

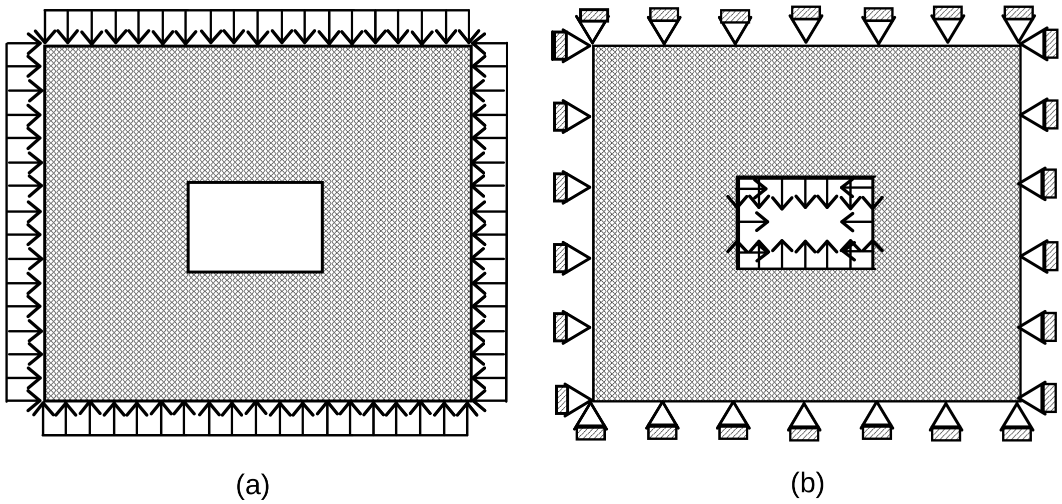

Figure 2 - Uniform loading of underground structures; (a) loading on the external boundary, (b) loading on the internal boundary

Vertical stress at mid-height of the pillar and along the centre-line in the roof as well as vertical displacement on the horizontal line in the roof are given in Figures 4 to 6. Regarding these results, the following conclusions may be made:

(a) The Null Model shows the initial response of the region to the virgin stresses before any structure was made there. Stress state remains almost constant, but there are some deformations throughout the model.

(b) The External Model gives the total values of stress and displacement, including the initial response of the region to the virgin stresses (before constructing the opening) plus the disturbances which resulted from making the structure.

(c) The Internal Model gives the relative changes of stresses and displacements around the structure -from the initial condition (virgin state) to the final condition (disturbed state). In other words, the initial conditions of stress and displacement are taken as zero state; hence the results show the induced stress and displacement which resulted only from the excavation of the opening.

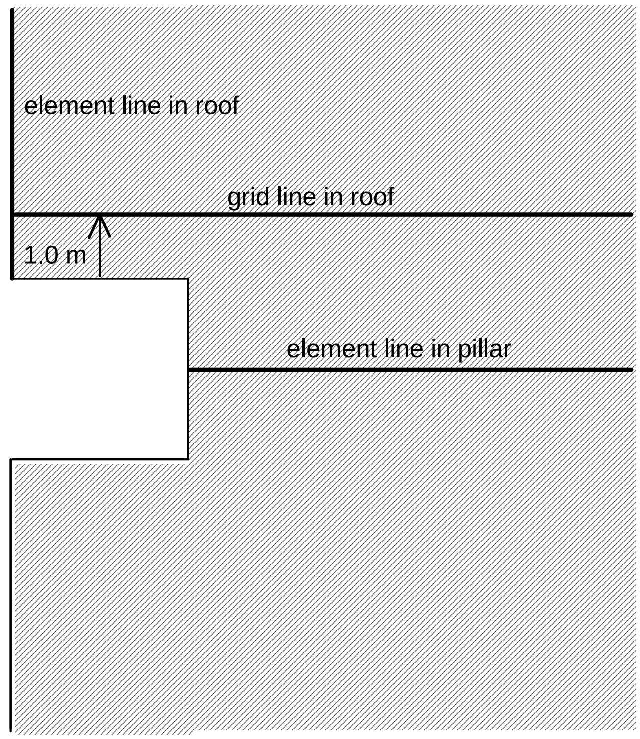

Figure 3 - Different locations in roof and pillar for analysing the results relating to loading techniques

These conclusions were exactly the same for all four loading conditions, and in summary the results suggest that:

i - The absolute values of stress around the structure can be obtained from either:
(a) External Models stress (σ_o), or
(b) Internal Model stress + Virgin stresses, ($\sigma_i + \sigma_v$).

ii - The relative displacement around the structure can be obtained from either:
(a) Internal Model displacement (D_i), or
(b) External Model displacement - Null Model displacement, (D_o-D_n).

where,

σ_o and σ_i	=	stresses which resulted from External and Internal Models, respectively
D_i, D_n and D_o	=	displacements which resulted from Internal, Null and External Models, respectively and
σ_v	=	virgin stress.

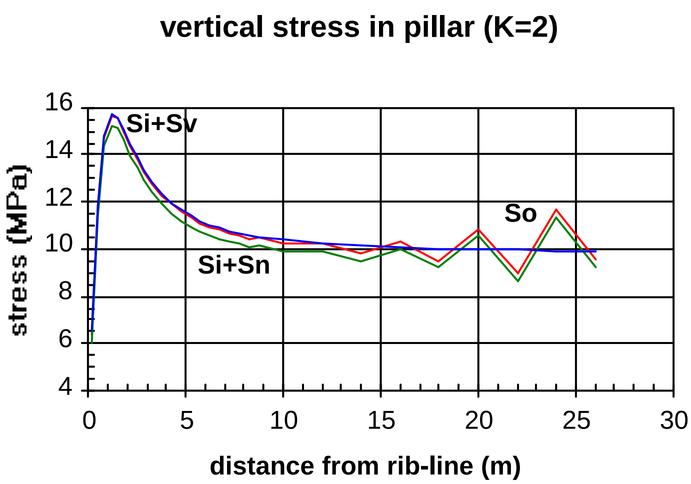

Figure 4 - Vertical stress at mid-height of the pillar

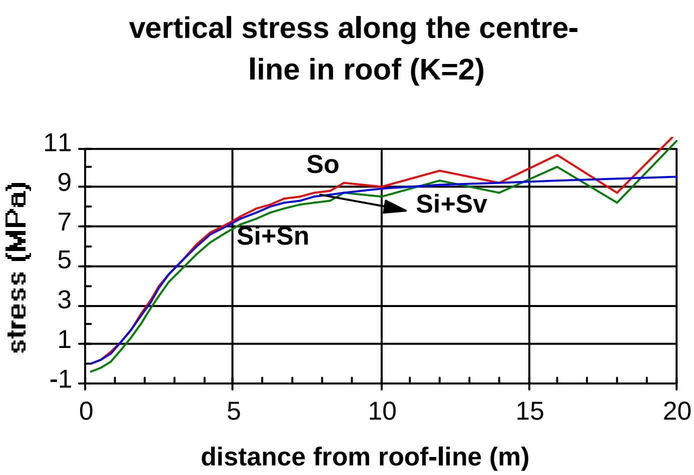

Figure 5-Vertical stress along centre-line in the roof

These results also indicate that the model size could be reduced by 50% using the internal loading technique without affecting the results, because there are no boundary effect on the results in this model. Although on such 2-D problems, this may not appear significant, in 3-D models the size reduction would significantly reduce the amount of required memory and computer running time.

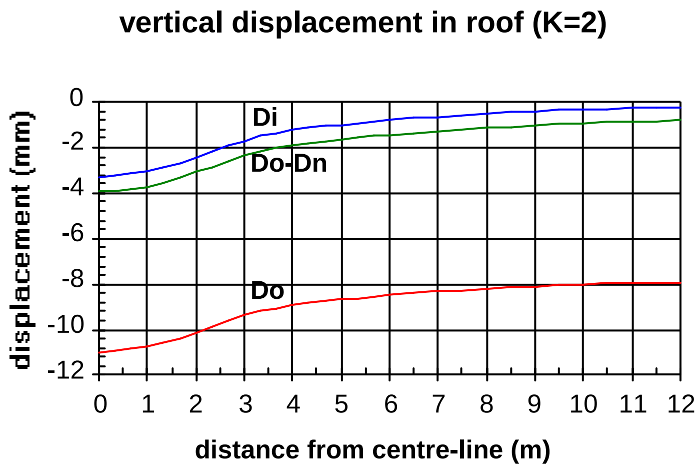

Figure 6-Vertical displacement on horizontal line in the roof

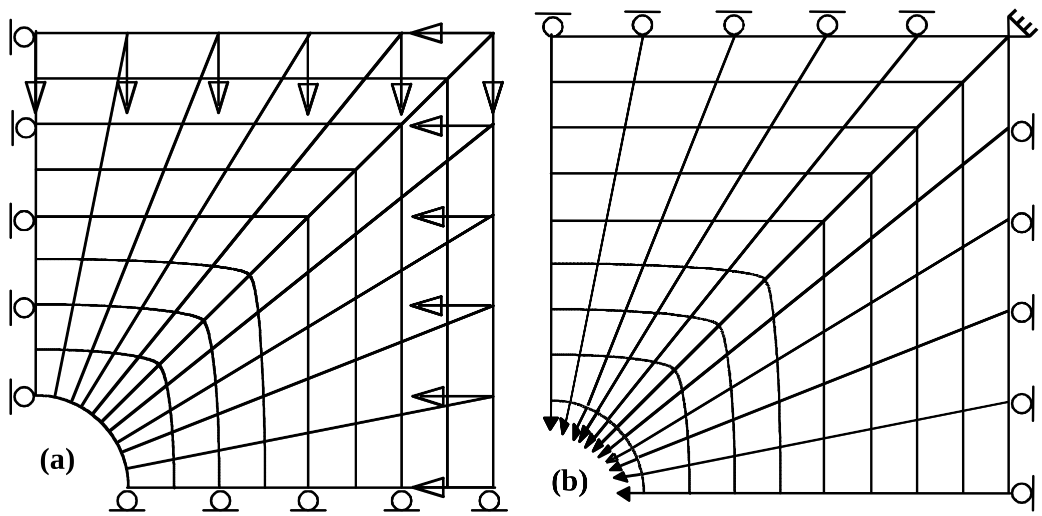

Figure 7 - A semi-infinite plate having a circular hole inside,
(a) External loading and (b) Internal loading models

3. VERIFICATION OF THE LOADING TECHNIQUES

In order to verify the results obtained from the Internal and External loading techniques, it was decided to apply both techniques to the classical problem of a semi-infinite plate having a circular hole inside (Figure 7). The results from the internally and externally loading models were compared with the results calculated using Krisch Solution (Obert and Duvall 1967). The comparison of the stress against distance from centre of the opening is given in Figure 8. It is shown that all three methods have similar results. If, however, the Krisch solution is accepted as the true values, it can be seen from Figure 9 that the internally loaded model consistently gives more accurate results. This is particularly true for points close to the opening - which would be the critical region in the case of underground openings. These results suggest that it is desirable to use internal loading when modelling underground structures. This applies to both accuracy of the results and computer costs.

The next point relating to the loading technique is the method of applying horizontal stress on the model. Since each stratum may have a different stiffness, it may seem that horizontal stress should be divided among the strata according to their stiffness. On the other hand, available in-situ measurements have shown a constant uniform distribution of horizontal stress for a limited range of depth. Therefore, two series of models were constructed and analysed to check this matter. In the first series (Uniform Model) a uniform horizontal stress was applied on all strata while in the second series (Stiffness Model) the horizontal stress was divided among the strata according to their stiffness. These two alternatives are structurally shown in Figure 10.

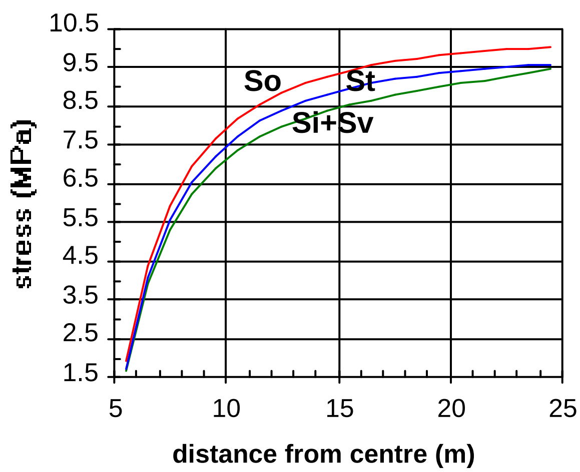

Figure 8-The comparison of the stress against distance from centre of the opening, S_o: External result; S_t: Theoretical result; S_i: Internal result; S_v: Virgin stress

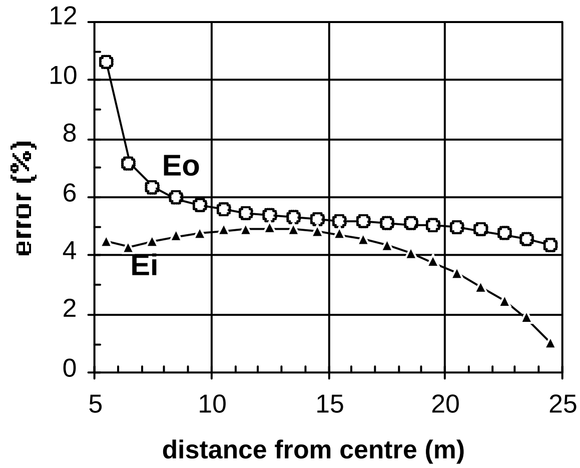

Figure 9-Accuracy of the External and Internal results against theoretical solution, E_o: Error of External loading; E_i: Error of Internal loading

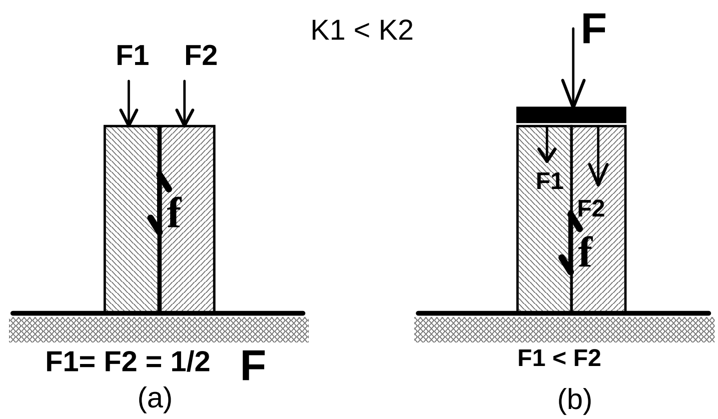

Figure 10-Uniform and Stiffness loading Models

Theoretical analysis shows that as long as the two parts in the model are bound together, there will be no difference in the average frictional force, f, calculated from either models, (a) or (b). This concept was checked by FE analysis. The results from FE models were in good agreement with the theoretical solutions. Figures 12, 13 and 14 show shear stress along three different lines in the model (Figure 11). Small discrepancy between the results from Uniform and Stiffness Models around the boundary of the model is due to the fact that the distribution of frictional force along the bedding planes are not exactly the same, but not important for the locations close to the structures, about 4 m from the rib-line.

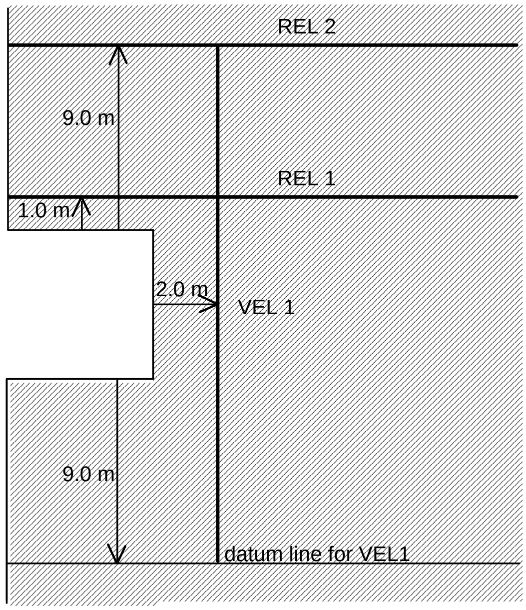

Figure 11-Different locations in the model for analysing the results relating to horizontal loading techniques

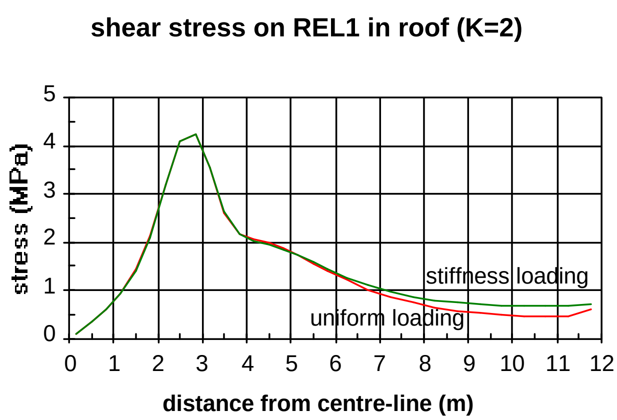

Figure 12-Shear stress along REL1 line in the roof

4.0 CONCLUSIONS

On the basis of the achievements from this investigation, it is proved that the loading technique (to simulate insitu stress)has a great influence on the results of any finite element analysis. Gravitational loading method is the appropriate method for modelling surface or shallow structures where it is possible to model the total cover (over- burden), or the gradient of the stress along the sides of the model is very small. The uniform loading method is suitable for modelling deep structures but it must be noticed that if the load is applied on the external boundary of the model, results are absolute magnitudes of stresses and displacements. For finding realistic displacements around the structure (relative values), the initial response of the model to the virgin stresses must be taken from the absolute values. On the other hand, if the load is applied on the internal boundary of the model, results are the relative values of the stresses and displacements, and for finding the actual value of stresses, the virgin stresses must be added to the relative values. However, the Internal Loading Technique has advantages, such as the results are more accurate, it is easy to add virgin stresses to model results to get the absolute magnitude of the stresses, it is possible to reduce the model size because there is no disturbance around the outer boundary of the model. There is not significance difference between applying of the horizontal stress uniformly or considering transitional zone.

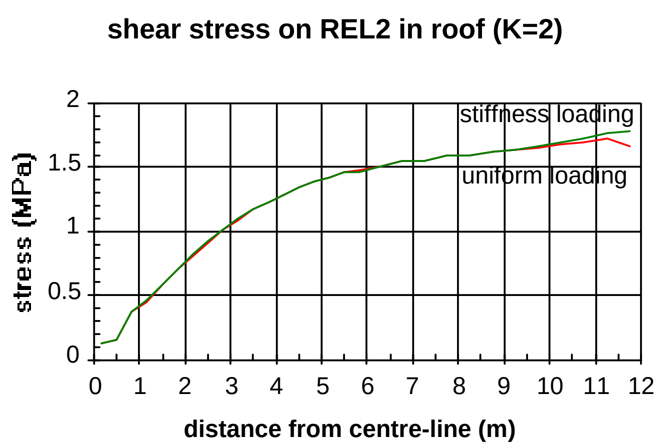

Figure 13-Shear stress along REL2 line in the roof

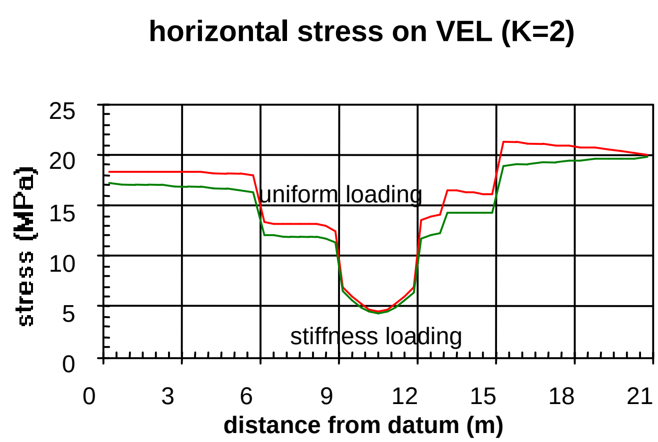

Figure 14-Shear stress along VEL1 line crossing strata vertically

According to the above achievements, the authors wish to recommend the Internal Model for deep underground structures. It will help to choose smaller model saving computer time significantly, and to obtain more accurate results.

Rerefernces

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